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ON THE RED END OF THE RED ARGON SPECTRUM.

By C. RUNGE.

THE wave-lengths of the red argon lines have been determined by W. Crookes,[1] by H. Kayser,[2] and by C. Runge and F. Paschen.[3] These determinations were all made visually. It is comparatively easy to measure the more refrangible red lines visually. But in the least refrangible part of the spectrum the sensitiveness of the eye decreases considerably and accurate measurements become difficult. Now I have lately obtained photographic plates that possess a maximum of sensitiveness somewhere about $\lambda = 7500$. They were manufactured and sensitized by Dr. Schleussner in Frankfurt. The relative intensities of the lines seem to be about the same as on plates sensitized with Alizarin blue S,[4] but I cannot say anything about the mode of preparation, as I was refused any further information. With these plates I have photographed the spectrum of argon as far

[1] W. CROOKES, *Chem. News*, **71**, 58. 1895.

[2] H. KAYSER, *Sitz. Ber. d. Berl. Akad.*, **24**, 1896; this JOURNAL, **4**, 1, 1896.

[3] C. RUNGE and F. PASCHEN, this JOURNAL, **8**, 99, 1898.

[4] See G. Higgs, *Photographic Atlas of the Normal Solar Spectrum*, Descriptive Supplement, p. 10.

as $\lambda = 8015$ in the first order of a concave grating of one meter radius. An exposure of several hours revealed even the weaker lines, which hitherto have not been measured. Photographs were taken of the red lines with and without the ultra-violet of the second order, in the latter case by cutting it off with a yellow screen. Only the strong argon lines 3947.647, 3949.107, and 4044.565 and the mercury line 4046.78 made their appearance through the yellow screen. For the wave-lengths of the ultra-violet argon lines I have taken the measurements of H. Kayser. The red lines do not appear as sharp as the ultra-violet lines. This is due, no doubt, to the action of the sensitizer, and lessens to some extent the accuracy of measurements.

For the illumination of the slit I have discarded the use of lenses when measuring the red lines by the method of coincidences with the second order; for I found that the difference of illumination by red and violet light caused by the difference of focal lengths may produce a considerable shift of the one order with respect to the other. This does not happen if the source of light is large, as in the case of a Bunsen burner or the electric arc or the Sun. But in the case of a vacuum tube placed end-on there is little light to spare, and if the image of the end of the capillary is not accurately focused on the slit, it may easily happen that the ruled surface of the grating is not fully illuminated, and this may cause a shift of the lines if the adjustment of the plate is slightly out. With lenses it is impossible to focus accurately for light of all wave-lengths. The use of concave mirrors instead of lenses does away with this source of error. The red and ultra-violet light must in this case follow the same path, except for differences from diffraction caused by a narrow slit, and the images of the slit must be accurately the same. There is no inconvenience in the use of two mirrors. It is possible to place them so that one compensates the astigmatism of the other; but an astigmatic image on the slit may even be an advantage.

The extreme red end of the spectrum seems to deserve special interest as the lines in this region are generally sparse. Thus

it becomes comparatively easy to identify a substance by some of its lines in this region. Argon, for instance, may be readily identified by its red lines, in a vacuum tube containing a little unprepared air, and it is not to the credit of spectroscopists that they suffered argon to remain undetected so long.

Wave-length	Intensity [1]		Number of determinations	Mean error	Previous Determinations		
	I	II			C. Runge and F. Paschen	H. Kayser	W. Crookes
7207.20	<1	1	2	0.13			
7273.13	3	6	3	0.02	7~~3~~73.04	7271.6	7263
7311.80	<1	1	2	0.05			
7316.15	<1	—	1	——			
7353.42	1	1	3	0.06			
7372.28	1	1	3	0.01			
7384.18	5	5	3	0.08	7384.22	7383.9	7377
7435.77	1	1	3	0.18			
7504.04	8	6	3	0.04	7504.5	7503.4	7506
7514.77	4	2	3	0.04	7515.4	7515.1	
7635.19	3	3	3	0.08	7636.2	7635.6	7646
7724.15	2	2	3	0.07	7725	7723.4	
7948.32	1	<1	2	0.05	7952		
8006.00	1	<1	2	0.04			
8014.73	1	<1	2	0.19			

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TECHNISCHE HOCHSCHULE, HANNOVER,
February 1899.

[1] The intensities are given first as they appear on the photographic plate (I), and secondly, as they appear visually (II). The numbers are mere estimations and have little value beyond signifying that one line appears stronger or weaker than another, larger number meaning greater intensity. But however rough, the numbers show that the sensitiveness of the plate has a maximum in this region.

THE ATMOSPHERE OF VENUS.

By Henry Norris Russell.

It has been known for more than a century that, when Venus is near inferior conjunction, the cusps of her crescent project beyond the position which they would occupy were she merely an opaque sphere like the Moon, so that more than half her circumference is visible.

This phenomenon was first noticed in 1790, by Schroeter, who made numerous measures of its extent in 1793 and 1794.[1] He observed the planet low in the twilight just after sunset or before sunrise, and distinguished a faint bluish light outlining the circumference for some distance beyond the apparent bright cusps, which, as his measurements showed, also projected somewhat beyond their geometrical position. As later observers have all, so far as I can learn, worked in the daytime, it is not surprising that only the bright part of the cusps seems to have been seen by them, while the faint bluish light was lost in the glare of the sky.

The next observations are those of Mädler,[2] in 1849. He succeeded in seeing fully 240° of the circumference of Venus, but was in this respect far surpassed by Lyman, who in 1866,[3] and again in 1874,[4] succeeded in observing the planet several times when she was within $1\frac{1}{2}°$ of the Sun's limb, when the cusps had coalesced and she appeared as a luminous ring. I can find no account of such observations during the next favorable conjunction in 1882, but in 1890 Barnard,[5] at the Lick Observatory, saw 340° of the circle, and would certainly have seen the whole, had not the days of closest approach been cloudy.

[1] For a very full account see his *Aphroditographische Fragmente*, pp. 90 ff.

[2] *Astronomische Nachrichten*, **29**, 107.

[3] *American Journal of Science* (2), **43**, 129.

[4] *Ibid.* (3), **9**, 47.

[5] *Astronomische Nachrichten*, **126**, 295.

The following observations were made by me at the Halsted Observatory of Princeton University in the winter of 1898. The telescope used was the 5-inch finder of the great equatorial. Its object-glass was screened from direct sunlight by the sunshade which is fitted to the tube of the large telescope near its object-glass for use during spectroscopic work on the Sun. A hole was cut in this just large enough for the finder to look through, and its object-glass was thus completely shaded when it was pointed at an object 1° from the Sun's center.

November 29, 1898. Clear day with good definition.

Diameter of Venus, 2.002 micrometer revolutions (mean of four measures).

Distance between lines tangent to cusps and to opposite limb of planet, 1.63 revolutions (mean of three measures).

Hence: Total visible arc, 254°. Each cusp projects 37° beyond its theoretical place.

December 2, 1898, 10:30 A.M. Venus being about 1°45′ from the Sun's center, I found, after my eye had become accustomed to the brightness of the field, that the complete circle of the planet could be seen by glimpses. I informed Professor Young of the fact, and he found that by stopping out the edges of the field-lens of the eyepiece the fainter portion of the ring might be seen almost steadily. Venus appeared to both of us as a ring of light, very much brighter on the side toward the Sun. The faintest part of the ring was directly opposite the Sun, and was barely visible. No considerable irregularities were visible on the ring, and no coloration was noticed. At no time during the observations was the unilluminated part of Venus seen.

Cloudy weather prevented further observation till December 7, when the planet had moved far enough from the Sun to be observable with the 23-inch telescope, with which the following measures were made:

Diameter of Venus, 6.783 revolutions (mean of three).

Distance between tangents to cusps and limb, 4.055 revolutions (mean of four).

Hence: Visible arc, 202.6°. Each cusp projects 11.3°.

The necessary inference from the observations is that, for some reason, more than half of Venus' surface must be illuminated by the Sun. It is true that since the Sun, as seen from Venus, has a diameter of 44′, a strip of her surface extending 22′ of arc (measured on her surface) beyond the geometrical terminator, must receive light from a part of the Sun's disk; but this penumbral illumination is not nearly enough to account for the observed phenomena. The cause of this extension of the illuminated area has always been supposed to be atmospheric, since it is impossible to see how more than half an opaque globe without atmosphere can either be lighted by the Sun, or seen by us at any one time.

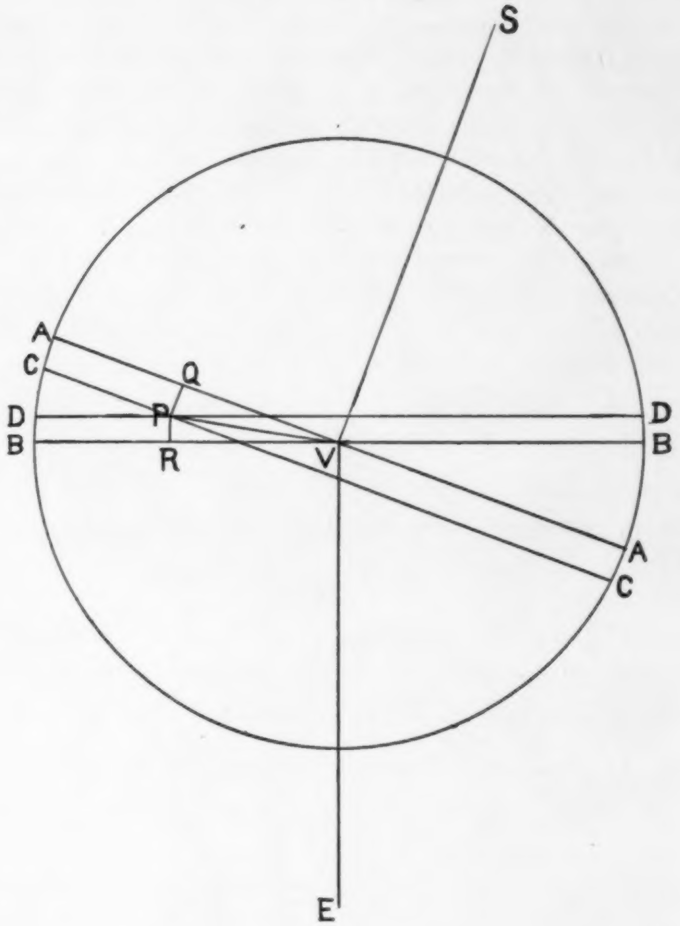
An atmosphere, both by its refraction and by the illumination of its upper layers by light which had grazed the planet's surface, would extend the directly sunlit area beyond the edge of that part which, without the intervention of the atmosphere, would be illuminated by at least a part of the Sun's disk, by a strip whose width would depend upon the atmospheric height and density.

And since, even if we consider atmospheric refraction and illumination, the part of the planet visible to us is that part which would be illuminated by the observer, were he a luminous point, the portion of her surface visible to us would be extended by atmospheric action by a strip of the same width.

To determine the relation between the width of these strips and the amount of prolongation of the cusps, let s be the width of one of the strips, measured in arc on the planet's surface. Consider the triangle formed by Venus, the Earth, and the Sun, and let a be the exterior angle at Venus. Finally let p be the arc by which one of the cusps is observed to project beyond the position it would occupy if there were no atmosphere.

Consider the projections of the illuminated and visible parts of her surface on the plane passing through Venus, the Sun and and Earth, as represented in the annexed figure. In the figure V represents the center of Venus, and is also the projection of the position of the geometrical cusp. VS is the line to the Sun,

VE that to the Earth. *AA*, which is perpendicular to *VS*, is the position of the geometrical terminator (assuming the Sun to be a luminous point), and *BB* is the projection of the geometrical



limb. The angle *AVB* is the supplement of *EVS*, and is therefore equal to α. The apparent terminator will be a small circle on Venus' surface, whose distance from *AA* is $s + 22'$ (the $22'$ is the allowance for the Sun's semidiameter). Its projection is

CC. The apparent limb will be a small circle whose distance from *BB* is s. Its projection is *DD*. *P*, the intersection of *CC* and *DD*, will be the projection of the apparent cusp. Drawing *PQ* and *PR* perpendicular to *AA* and *BB* respectively, we have $PQ = s + 22'$ and $PR = s$. The distance *VP*, which is the projection of the arc p, might be exactly calculated from these data by Trigonometry, but since, owing to the difficulties of observation, p cannot be very accurately determined, we may obtain a much simpler approximate formula, abundantly accurate for all practical purposes, by supposing that $PQ = PR = s + 11'$. This will give us a value of *VP* which is in error only by a quantity of the second order, negligible with respect to s.

We now have the triangles *QVP* and *RVP* equal in all respects. The angles at *V* are each equal to $\frac{1}{2}\,a$, and in the spherical triangle of which *PVQ* is the projection we have $\sin PQ = \sin PV \sin PVQ$, or

$$\sin(s + 11') = \sin p \sin \tfrac{1}{2}\,a \tag{1}$$

which is the desired formula. In most cases it is a safe approximation to substitute the small arcs involved for their sines, which gives us

$$s + 11' = \tfrac{1}{2}\,a \sin p. \tag{2}$$

If we are given the elongation v of Venus, as seen from the Earth, instead of the angle a at Venus, we have in the Sun-Venus-Earth triangle, $\frac{\sin v}{\sin a} = \frac{\rho}{r}$, where r and ρ are the radii vectores of the Earth and Venus respectively. This may be written approximately $\frac{v}{a} = \frac{\rho}{r}$. Substituting in (2) we have

$$s + 11' = \frac{r}{2\rho}\, v \sin p. \tag{3}$$

Schroeter, in reducing his observations, used a formula which gives the whole distance between the geometrical terminator and limb at the apparent cusps, and made no allowance for the Sun's semidiameter. Mädler inadvertently introduced the angle at the Earth into his formulae, instead of that at Venus, and Lyman

followed him in this mistake. Neison[1] points out this error, and is the first to give the correct formulae in substantially the form developed above, but his numerical results for s are all $11'$ too small—probably because he used the Sun's diameter in place of its radius.

It has, therefore, seemed worth while to reduce anew all accessible observations by the correct formula.

The results are as follows :

OBSERVATIONS OF EXTENT OF FAINT LIGHT BEYOND APPARENT BRIGHT CUSPS.

Observer	Date	a	p	s
Schroeter - - - -	1790, Mar. 12	17° 43′	15° 19′	2° 20′*
" - - - -	1793, May 21	13 55	19 28	2 19
" - - - -	1794, Dec. 20	26 30	11 56	2 27
			Mean	2° 22′
			Extent of brighter portion	59′

*These are really differential measures, and therefore the correction for the Sun's semidiameter has not been applied to them.

NIGHT OBSERVATIONS OF TOTAL EXTENT OF VISIBLE CUSPS

Observer	Date	a	p	s
Schroeter				
7-foot telescope power 74 - -	1794, Dec. 22	22° 49′	13° 58′	2° 33′
	1795, Jan. 10	17 42	18 9	2 34
Power 160 - -	1794, Dec. 15	35 45	12 10	3 31
	Dec. 17	32 14	12 22	3 14
	Dec. 18	30 19	15 15	3 46
	Dec. 19	28 26	14 29	3 20
13-foot telescope power 136 - -	1794, Dec. 20	26 30	17 23	3 44
	Dec. 22	22 49	20 22	3 46
	Dec. 23	20 22	22 7	3 38
			Mean	3° 21′

DAY OBSERVATIONS OF TOTAL EXTENT OF VISIBLE CUSPS

Observer	Date	a	p	s
W. Herschel - - -	1793, May 20	16° 14′	15° 0′	1° 54′
" - - - -	1793, May 20	16 14	24 7	3 12

[1] *Monthly Notices*, **36**, 348.

Observer	Date	v	p	s
Mädler - -	1849, May 9,	6° 16.7′	10°	35′ *
" - -	10,	5 7.8	22½	71
" - -	11,	3 57.1	27½	65
" - -	11,	3 36.5	27½	59
" - -	12,	3 25.7	30	61
" - -	12,	3 25.7	30	61
" - -	12,	6 26.4	30	61
" - -	15,	6 25.4	17½	69
" - -	16,	7 36.9	15	71
				Mean 65.3′
Lyman - -	1866, Dec. 7,	6° 25′	20°	79′
" - -	10,	1 24	90	> 46.4
" - -	12,	1 52	90	> 65.5
" - -	12,	2 4	90	> 74
" - -	14,	5 6	25	77
" - -	15,	6 43	15	60
" - -	18,	11 23	11	78
				Mean 74′
" - -	1874, Dec. 8,	32.5′	90°	> 11.1′
" - -	10,	2° 31.7	49 41′	68.1
" - -	11,	4 2.5	26 38	63.3
" - -	11,	4 20.4	25 43	66.7
" - -	12,	5 58.3	17 41	63.4
				Mean 65.4′
Barnard - -	1890, Dec. 1,	5° 28′	45°	105′
" - -	5,	3 2.8	80	79
				Mean 92′
Russell - -	1898, Nov. 29,	4° 45.7′	37°	75′
" - -	Dec. 2,	2 30.0	90	> 64
" - -	7,	13 11	11.3	66
				Mean 70′

* Mädler states that the object-glass was not sufficiently shielded at the time of taking this measure. It is excluded in taking the mean.

The results of the different series agree well enough to show that the projection of the cusps visible in an ordinary telescope in the daytime is about that which results from the formula when

we put in it $s = 70'$. The only seriously divergent observations are Barnard's; and it is very probable that he may have seen more than an ordinary observer could.

If we explain this enlargement of the illuminated and visible parts of Venus' surface by the refraction of her atmosphere (as has usually been done), it is evident that the width of the strip s measures the horizontal refraction r_0, which, in consequence, must be some 68 or 70 minutes. This is almost exactly twice that of our atmosphere, and to produce it would require an atmosphere more than twice as dense or extensive as ours, as the force of gravity on Venus' surface is about four fifths of that on the Earth's. The height of such an atmosphere, if of composition similar to ours, would be about fifty-five miles as against forty for us.

But it may easily be shown that, if the horizontal refraction of Venus' atmosphere is great enough to account, alone, for the observed prolongation of the cusps, a very conspicuous refracted image of the Sun ought to be visible on the limb of Venus farthest from the Sun, when she appears as a luminous ring.

Consider the paths of rays supposed to be emitted from the Earth, and refracted to the maximum amount by Venus' atmosphere. They would graze her surface and then pass on, being deviated in all by twice the horizontal refraction r_0, and would form a cone with its vertex about 200,000 miles behind Venus, a distance which is negligible in comparison with the Sun's distance. The trace of this cone on the celestial sphere is a circle of radius $2r_0$, whose center is the apparent geocentric place of Venus. Any part of the Sun which, as seen from Venus (or, strictly speaking, from the vertex of the cone), appears to encroach on this circle, will be visible to us by refraction through Venus' atmosphere, as a luminous arc along her limb. If we draw lines through the center of the circle to pass through its points of intersection with the Sun's limb, or, if more than half the Sun is inside the circle, to touch his limb, then the angle between these lines will clearly measure that arc of Venus' limb on which the Sun's refracted image will be visible from the

Earth, since rays falling outside these lines cannot meet the Sun. The radius of the circle is $2r_0$, and that of the Sun, as seen from Venus, is $22'$, while the distance of their centers is obviously a. If these are known, it is an easy trigonometrical problem to calculate the extent of the Sun's image along Venus' limb. If we assume $r_0 = 68'$ we find the following:

a	Extent of image	a	Extent of image
$>158'$	$0°$	$80'$	$32°$
154	10	60	43
150	13½	40	67
140	18	30	94
120	21	22	180
100	26	<22	360

The middle of this image must evidently coincide with the point of Venus' limb farthest from the Sun.

Now if Venus' atmosphere were perfectly transparent, the intrinsic brightness of the refracted image would be that of the Sun's surface itself, and, though its apparent area would be very small, its great brilliance would make it a very conspicuous object. As an example of this, let us discuss the case when $a = 150'$, $r_0 = 68'$, which are very closely the circumstances of my observation of December 2, 1898.

Light from the Sun's limb nearest Venus, to reach us by refraction through her atmosphere, must in this case be deviated by at least $128'$. The top of the Sun's refracted image would, therefore, be at that height above the planet's limb at which the horizontal refraction is reduced from $68'$ to $64'$. But the horizontal refraction of an atmosphere varies as its density, and we know that the density of our atmosphere at a height of 3.57 miles is half that at the Earth's surface. Since the force of gravity on Venus is 0.82 of ours, the height above her surface where the density is reduced one half must be about four miles. The height at which the horizontal refraction would be reduced to $64'$ comes out 0.35 mile. This, then, would be the actual width of the widest part of the Sun's image. Now when Venus is in inferior conjunction one mile on her surface subtends $0.008''$ as seen from the Earth. The apparent width of the Sun's

image would then be 0.003″ at its widest part. It extends over $13\frac{1}{2}°$ of Venus' circumference, which, since Venus' diameter at this time is 64″, corresponds to a length of 7.5″. Its width will decrease more rapidly near its ends than toward the middle, and so its mean width will be about two thirds of its greatest width, that is, 0.002″. Its apparent area is then 0.015 square seconds of arc.

Now, according to our hypothesis, the width of the visible illuminated portion of Venus' surface, measured on her surface between the terminator and limb, varies from 5° 8′ on the side toward the Sun to 8′ on the opposite side. Its apparent width, being equal to the semi-diameter of Venus multiplied by the versed sine of its true width, varies from 0.128″ to 0.0002″. Its mean width is about 0.064″, and since its length — the circumference of Venus — is about 200″, its area is some 12.8 square seconds, or about 850 times that of the Sun's image. We may determine the relative brightness of equal areas of the surfaces of the Sun and Venus as follows: The Sun's apparent radius is about 960″. Its apparent area is about 3,000,000 square seconds. Venus at her greatest brightness is about 40″ in diameter, but only about one fourth of her disk is visible. So her apparent area is about 315 square seconds. The Sun's apparent area is thus about 9500 times Venus'. But the Sun is of the $-26\frac{1}{2}$ magnitude on the ordinary scale, and Venus is never as bright as the -4th magnitude. The difference is $22\frac{1}{2}$ magnitudes, which means that the Sun is 1,000,000,000 times as bright as Venus at her greatest brightness. But he appears to us rather less than 10,000 times as large. Therefore, a given area of his surface must be at least 100,000 times as bright as the same area of hers. The Sun's refracted image, which is, on our supposition, as bright intrinsically as his surface, must therefore be about 120 times as bright as all the rest of the luminous ring taken together, and $\frac{0.003'' \times 100{,}000}{0.128''}$ or more than 2000 times as bright as an equally long piece of the opposite side of the ring.

Now, on the second of last December, it was noted both by Professor Young and by myself that the faintest part of the ring was just opposite the Sun, in just the place, as it now appears, where the refracted image should have appeared. This portion of the ring was scarcely visible, and not nearly as bright as an equal part of the opposite side of the ring. It was, therefore, not more than $\frac{1}{2000}$ as bright as it should have been on the hypothesis that Venus has a perfectly transparent atmosphere with a horizontal refraction of $68'$.

In the case of Lyman's observation of December 8, 1874, when a was only $45'$, the width of the widest part of the solar image would on this hypothesis be $0.050''$, and that of the widest part of the true crescent $0.057''$. The side on which the solar image was should have been some 80,000 times as bright as the other. Lyman expressly states that it was fainter than the crescent proper.

There are two things, however, which might greatly diminish the contrast between the Sun's image and the opposite part of the crescent. These are specular reflection of Venus' surface and absorption of light in her atmosphere. If the first of these occurred to any marked extent, a bright reflected image of the Sun, of the same extent along the limb as the refracted image, would be formed on the side of Venus next the Sun. Although it has been contended by Brett that there are indications of such a reflected image, the weight of evidence is strongly against it, and so we are not at liberty to explain the difficulty by this hypothesis.

As for the hypothesis of absorption, since light reflected to us by Venus' surface has even a longer path through her atmosphere than the refracted light, the contrast between the solar image and the neighboring parts of the limb would be as great as ever, no matter how great the absorption, and it would be impossible for the region where the Sun's image ought to be, to be the faintest part of the whole ring.

If we assume that Venus' atmosphere scatters so much of the light passing through it that there is very little apparent

difference in brightness between the Sun's limb and the surrounding illuminated air, and at the same time absorbs considerably the light passing through it, we avoid the last-mentioned difficulty, although we must suppose the absorption to be great in order to reduce the brightness of the side remote from the Sun to its observed value. But in this case the illuminated atmosphere would form a luminous ring around the planet long before she was near enough to the Sun to bring his refracted image into view. So we see that in any case the theory which ascribes the whole of the observed prolongation of the cusps to horizontal refraction is untenable.

In fact, in the case of Lyman's observation above mentioned, if the horizonal refraction had been greater than $12'$ and the atmosphere clear, the Sun's image would certainly have appeared as a brilliant arc some 20° in length. Lyman's description of what he saw is: "The ring was brightest on the side toward the Sun—the crescent proper. On the opposite side the thread of light was fainter, and of a slightly yellowish tinge." It is evident that he did not see any such image of the Sun as is described above.

Even if Venus' atmosphere were very hazy, we shall soon see that the observed prolongation of the cusps would be greater than it actually is, if the haze rose to a height of much more than 4000 feet above the surface. The atmosphere above this level must be relatively clear, and if the horizontal refraction at this level had been more than $12'$ the Sun's refracted image would have been visible. This would make the refraction at the visible surface of the planet $14'$, and this is the maximum value consistent with Lyman's observation. It corresponds to an atmospheric density about two fifths of our own.

The phenomena preceding internal contact at the transits of Venus, when the part of her disk outside the Sun is surrounded by a line of light bright enough to be seen through the solar eyepiece, are undoubtedly due to the refraction of her atmosphere. Dr. C. S. Hastings has shown[1] that a satisfactory explanation

[1] *Sidereal Messenger*, 1, 273.

of the observations can be found in the refracted image of the Sun produced by the rare upper layers of her atmosphere. The total deviation of the light is in this case, however, only a very few minutes of arc, and the presence or absence of denser and more strongly refracting layers in the atmosphere would be without effect upon this phenomenon.

Under the circumstances, therefore, we see no more probable explanation of the phenomena than the hypothesis that the atmosphere of Venus, like our own, contains suspended particles of dust or fog of some sort, and that what we see is the upper part of this hazy atmosphere, illuminated by rays that have passed close to the planet's surface. This explanation makes the phenomenon exactly analogous to our own twilight. Consider a particle of haze at a height h above Venus' surface. Its horizon will be at a distance t, between which and h will exist the relation $h = r\,(\sec t - 1)$, where r is the radius of Venus, 3850 miles. If the distance of the particle beyond the geometrical terminator is less than $t + 22'$, it will be lighted by some part of the Sun, and if its distance beyond the geometrical limb is less than t, it will be visible from the Earth. From this it follows at once that an envelope of haze around Venus of height h would extend the illuminated and visible areas by strips of constant width in just the way described in the first part of this article, and that the width of the strips (barring the penumbra) would be t.

If refraction also occurred, it would widen the strip still further by its amount r_0, and we should have for the total width $s = r_0 + t$. Now we know that for the brighter portion of the extension of the cusps $s = 68'$ approximately; and we have proved that r_0 is not greater than $14'$. If we set $r_0 = 0$, then $t = 68'$, and we find h to be about 4100 feet. If $r_0 = 14'$, $t = 54'$, and h is 2600 feet. Between these limits must lie the height of that hazy lower part of the atmosphere of Venus which sends us light enough to be visible through the brilliantly illuminated foreground of our own atmosphere. For the fainter light seen by Schroeter $s = 3°21'$ and h is $6\frac{2}{3}$ miles. The true extent of

the twilight, as Schroeter remarks, must be greater, since the part measured was bright enough to be seen through our own strong evening twilight. The full height of her atmosphere is probably much more than seven miles, and may well be twenty or thirty miles. To this last-mentioned height would correspond a maximum density of perhaps one tenth that of our atmosphere at sea level (supposing her atmosphere to be of the same composition as ours). This means of course the density of Venus' atmosphere at her apparent opaque surface. If this is a cloud-layer, or a layer of atmosphere so hazy as to appear opaque when viewed obliquely, the density at the surface of her (supposable) solid crust may be very much greater; but we can know nothing about it.

The amount of haze in Venus' atmosphere would naturally decrease at higher levels. If this change were uniformly gradual it seems hardly probable that the results obtained by so many observers, with very different instruments and conditions of observation, would have been in as good general agreement as is actually the case. It is therefore likely that there is a more or less abrupt decrease in the haziness of Venus' atmosphere at a height of about 4000 feet above her apparent surface. The transition between the lower hazy and upper clear layers is probably not sharp, and this accounts perfectly for the divergence shown by some of the results for s — notably Barnard's — and also for the smaller values of s usually found when Venus is nearer the Sun and, therefore, seen through a brighter sky.

The faintness of a small portion of the ring, which was noticed by Lyman in 1866 and again in 1874, may be simply explained by supposing a part of the 4000-foot haze-bank to be cut off by low mountains.

A strong confirmation of the theory that Venus' atmosphere is less extensive than the Earth's is found in the spectroscopic observations of her atmosphere. These show, somewhat doubtfully perhaps, the presence of water vapor and probably of oxygen, but in small quantities only, the telluric atmospheric lines being only slightly strengthened. The white, or at most

faintly yellowish tinge of the twilight side of the ring phase of Venus points the same way. The color of our sunset sky shows that the Earth's twilight band, viewed under the same circumstances, would be yellow, or even red.

To recapitulate:

(1) The observed prolongation of her cusps shows that the sunlit and visible areas on Venus extend about $1^\circ 10'$ farther than they would on an opaque sphere without atmosphere.

(2) This has usually been explained as the result of the refraction of a clear atmosphere, more than twice as extensive as our own; but a consequence of this theory is that, when Venus appears as a luminous ring, a very conspicuous refracted image of the Sun ought to appear on that part of the ring farthest from the Sun; and this image has never been seen, even when a refraction of only $12'$ would have produced it; nor will atmospheric absorption or haziness explain its absence consistently with the visibility of the complete ring unless we assume that the horizontal refraction at Venus' apparent opaque surface is less than $14'$; while a much smaller amount of refraction will explain the transit phenomena satisfactorily.

(3) The observed prolongation of the cusps can be explained as due to twilight illumination of Venus' atmosphere, on which hypothesis the height of that part of her atmosphere which is bright enough to be seen through our own illuminated atmosphere in the daytime is about 4000 feet. Schroeter's observations show that the part of the atmosphere which can be seen through our evening twilight (and is probably less hazy than the previously mentioned part), is six or seven miles high. The total height of the atmosphere must be much greater. The observed irregularities of the luminous ring may be explained on this hypothesis by the presence of relatively low mountains or clouds on Venus.

(4) We may conclude that there is no satisfactory evidence that the atmosphere of Venus is more than one third as dense or extensive as the Earth's, and that it is almost certain that no light reaches us which has been deviated by refraction through

more than 28′. If there are any denser parts of the atmosphere they must be so hazy, or absorb so strongly, that when seen horizontally they seem opaque. The spectroscopic observations of the absorption of Venus' atmosphere, and the color of her twilight ring, both fall in with the theory of its small density. Moreover, Venus' surface temperature is probably higher than the Earth's, and the correspondingly higher molecular velocities of gases, together with the smaller force of gravity, lead us to expect an atmosphere of small extent and density.

PRINCETON UNIVERSITY,
April 8, 1899.

A PHOTOMETRIC METHOD FOR THE DETERMINATION OF THE EXPONENTIAL CONSTANT OF THE EMISSION FUNCTION.[1]

By F. PASCHEN and H. WANNER.

THE law expressing the dependence of the intensity of the radiation J upon the absolute temperature T and the wave-length λ of "the absolutely black body" has, according to the theory of W. Wien,[2] the form:

$$J = c_1 \lambda^{-5} e^{-\frac{c_2}{\lambda T}} \cdot \qquad (1)$$

The correctness of this law has been rendered probable by the measurements of one of us on the energy spectra of different surfaces.[3] A better confirmation of the law has been given by more recent experiments with a source of radiation approaching closely to the ideal black body, as to which a report will be made later.

Supposing that formula (1) expresses the correct law, it would be important to determine accurately its two constants, c_1 and c_2. We shall describe here a photometric method which seems well adapted for the very accurate determination of the constant c_2.

The basis of the method is that the change of intensity of a limited spectral region of mean wave-length λ corresponding to a change of temperature is given solely by the constant c_2. Introducing Briggs' logarithms formula (1) gives for this case

$$\log J = \gamma_1 - \gamma_2 \frac{1}{T}, \qquad (2)$$

where

$$\gamma_1 = \log (c_1 \lambda^{-5}) \ldots (2_a) \text{ and } \gamma_2 = \frac{c_2}{\lambda} \log e \ldots\ldots \qquad (2b)$$

[1] *Sitzungsberichte der Akademie der Wissenschaften zu Berlin*, 1899, II. Session of January 12.

[2] *Wied. Ann.*, **58**, 662, 1896. *Sitzungsberichte*, 1893, p. 55.

[3] F. PASCHEN, *Wied. Ann.*, **60**, 662, 1897.

The corresponding curve is designated as an isochromatic line in the paper by Paschen already cited. It is a straight line if $\frac{1}{T}$ is made the abscissa and $\log J$ the ordinate. To get the constant c_2 it is only necessary to determine γ_2, *i.e.*, the inclination of the straight line. Since this determination reduces to that of the ratio of two intensities of equal wave-length, it does not presuppose an apparatus for measuring radiation, which registers all the radiation which strikes it. It suffices if the apparatus always indicates the same fraction of the intensity of the wave-length λ. For this any kind of spectral photometer would be suitable. Illuminate one of its slits from a constant source and the other with the light of the black body, and then in any spectral region, limited as narrowly as possible, whose mean wave-length is to be measured, observe the ratio of the intensity of the black body to that of the constant comparison light at two different temperatures. From this we get the ratio of the intensities J_1, and J_2 of the black body corresponding to the two temperatures T_1 and T_2. Determine γ_2 from the relation

$$\log \frac{J_1}{J_2} = \gamma_2 \left(\frac{1}{T_2} - \frac{1}{T_1} \right), \tag{2c}$$

which follows from formula (2), and find the value of c_2 from (2 *b*).

The superiority of this photometric method over the bolometric is due to the greater sensitiveness of the human eye for radiation of visible wave-lengths than of the bolometer, so that it is possible to carry out the observations with a comparatively narrow slit, and hence in a quite pure spectrum. According to Paschen's experiments, the purity of the spectrum is important for the measurement of isochromatic lines. A disadvantage of the method is that temperatures comparatively high, and hence difficult to measure, must be employed in order to get light of sufficient brightness in the visible spectrum. Although we had at our disposal for this purpose only a rather weak photometer optically, and although on the other hand we were not in a

position to measure very high temperatures with sufficient accuracy, we have nevertheless attempted to test the utility of this method for an average range of temperatures.

A König spectral photometer, the slit and ocular diaphragm of which were made as narrow as possible, served for the experiments. The experimental light was to approximate that of the absolutely black body as closely as possible. It was emitted by a surface of about 6 sq. mm, blackened with oxide of iron, and uniformly incandescent, the middle point of which was situated exactly at the center of a reflecting sphere of 15 cm diameter, that hemisphere only being employed which could receive light from the radiating surface. Opposite the radiating surface was a narrow aperture through which the radiation fell on the slit. According to Paschen the radiation of the absolutely black body is given out by the surface at the center, if the reflecting surface performs perfectly. The hollow hemisphere was of bronze, well polished, and projected fairly good images of objects at its center, which seemed to fall at the same place, from whatever portions of the spherical surface they might have been reflected. The radiating surface was the middle part of a platinum strip of 0.2 mm thickness, 4 cm length, and 7 mm breadth, obtained by folding a sheet of 0.1 mm thickness, and 14 mm breadth, and it radiated with uniform brightness, being heated by an electric current. The junction of a thermo-element of platinum and platinum-rhodium wires of 0.15 mm thickness was placed between the two platinum strips, pressed close against them, but electrically insulated from them, and at the center of the part serving for the experiments. The junction was hammered flat, and the connecting wires were insulated for a sufficient space between the two parts of the strip so that the heat conduction could not affect the junction. The other junctions of the thermo-element lay in melting ice, as the calibration of the element, which was kindly carried out for us by Mr. Holborn, depended upon this arrangement. The thermo-electromotive forces were compensated with accumulators and these were compared with a Clark cell.

The comparison light was a ground-glass disk illuminated by an incandescent lamp as the window of a lantern. The incandescent lamp was fed by an accumulator, the current of which remained sufficiently constant during experiments lasting one or two hours.

In the following table of the results of our observations λ signifies the mean length in μ of the spectral region investigated, T the absolute temperature of the comparison light, and J the intensity of the radiation in units of that of the comparison light. The slit and the ocular diaphragm were always kept at the same width. We give for each measurement, under the heading "slit-width" the extent in μ of the spectrum lying within the ocular diaphragm. A value of c_2 was calculated for each wave-length by combination of pairs of values according to the above formula. These individual values received weights according to the distance of the points included in the calculation, and then gave the mean placed below the table.

$\lambda = 0.6678\,\mu$ (slit-width $= 0.0114\,\mu$).

Results of observations:

	1	2	3	4	5	6
Log. J.	0.12840–1	0.80122–1	0.71170	0.71560–1	0.28332	0.25078–1
T.	1135.3	1234.9	1405.1	1222.2	1322.9	1152.3

By combination of the different points the following values were calculated for c_2, with their weights:

Nos.	1 and 2	1 and 3	1 and 4	1 and 5	2 and 3	2 and 4
c_2	14563	14395	14221	14418	14273	13762
Wt.	1	3	2	1	2	1

Nos.	2 and 6	3 and 4	3 and 5	3 and 6	4 and 5	4 and 6	5 and 6
c_2	14581	14896	14382	14388	14017	14187	14401
Wt.	1	1	2	3	1	2	1

$\lambda = 0.6678\,\mu$ (slit-width $= 0.0069\,\mu$).

Results of observations.			Calculation.		
No.	log. J	T	Nos.	c_2	Wt.
1	0.29308–1	1165.7	1 and 2	14348	3
2	0.57570	1388.1	1 and 3	14283	3
3	0.57138	1386.6	4 and 2	14073	2
4	0.57578	1205.3.	4 and 3	14113	2

The second series was made on another day and with a different intensity of the comparison light than the first. As a mean of the differently weighted numbers we obtain, for wave-length 0.6678μ, $c_2 = 14322$, mean error $= 62$.

$\lambda = 0.5893\mu$ (slit-width $= 0.0060\mu$).

T	1183.7	1180.9	1271.6	1270.8	1176.4
Log. J	0.41558–1	0.40556–1	0.04804	0.03386	0.36542–1

T	1333.9	1177.9
Log. J	0.45220	0.39610–1

A second series with another comparison light:

T	1214.9	1391.1	1388.5	1203.8
log. J	0.30328–1	0.41744	0.40112	0.24866–1

A calculation in the same manner as for $\lambda = 0.6678\ \mu$ gave for c_2 similarly varying values, the mean of which is $c_2 = 14489$, mean error $= 74$.

$\lambda = 0.5016\mu$ (slit-width $= 0.0041\mu$).

T	1186.0	1316.5	1401.5	1399.1	1309.9	1191.6
log. J	0.50146–2	0.53278–1	0.15238	0.13086	0.52928–1	0.61174–2

With another comparison light:

T	1210.7	1376.6	1377.2	1203.8
log. J	0.96886–2	0.17810	0.22324	0.88402–2

Computation gives for c_2 the mean value 14467, mean error $= 143$.

$\lambda = 0.4861\ \mu$ (slit-width $= 0.004\ \mu$).

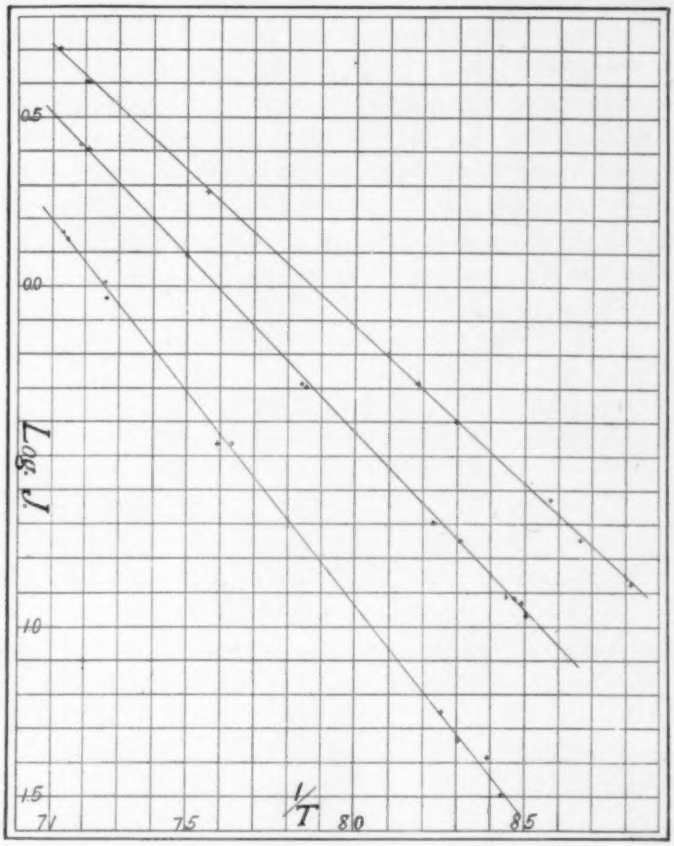
T	1242.5	1416.6	1415.7	1238.6
log. J	0.38658–1	0.66332	0.67458	0.36348–1

Computation gives as a mean $c_2 = 14473$, mean error $= 62$.

Summary of the values of c_2 obtained at a different wave-length:

λ	0.6678	0.5893	0.5016	0.4861	Final mean
C_2	14322	14489	14467	14473	14440
m. e.	62	74	143	62	

The figure shows the isochromatic lines for three wave-lengths with the observed points ($\log J$) as a function of $\frac{1}{T}$. We conclude that formula (1) is confirmed within the limits of possible errors, as far as it can be tested by our observations. First,



the isochromatic curve of each wave-length is a straight line, and second, the isochromatic lines of different wave-lengths give an equal value for c_2, within limit of possible errors of observation. The agreement is further extended, however, in a remarkable manner, for the value obtained is identical with that gotten

by an entirely different method with bolometric measurements. We have found as a mean value $c_2 = 14440$ ($\mu \times$ abs. temp.), and believe we must admit an uncertainty of some 2 per cent. The earlier measures of Paschen already cited (*loc. cit.*, p. 707) gave, for the different bodies investigated, values of the constant c_2 which lay between 15000 (for platinum) and 13700 (for carbon). The surmise was there expressed that the value for the absolutely black body would be about 14000.

The bolometric measurements on the wave-length of the energy-maximum at different temperatures,[1] recently made by one of us, gave for this same radiation on which our photometric measures were made results which led to the same value of the constant c_2. For example, the following values of the temperature and the corresponding wave-lengths λ_m of the energy-maximum were found.

Temp. C.	Temp. Abs.	λ_m (μ)	$\lambda_m \times$ T (Abs.)
1083.5	1356.5	2.138	2900
991.0	1264.0	2.293	2898
867.9	1140.9	2.537	2894
805.7	1078.7	2.674	2884
666.8	939.8	3.076	2891
523.3	796.3	3.605	2870
398.2	671.2	4.265	2862
195.7	468.7	6.026	2826

The radiation seems somewhat further from the ideal at lower temperatures than at higher, for the total radiation in the range from 100° to 400° C. increases by about 5 per cent. if the iron oxide in the reflecting sphere is replaced by a surface blackened with lampblack. The energy curves in this case gave

390.4° C.	663.4° Abs.	$\lambda_m = 4.355$	$\lambda_m \times T = 2889$
256.2	529.2	5.468	2894

hence the same value as for iron oxide in the reflecting shell at higher temperatures. As to the method of determining accurate

[1] The lampblacked bolometer strip employed was situated at the center of a small, accurately reflecting hemisphere, which repeatedly returned upon it the radiation it reflected, and hence thereby made it blacker (*loc. cit.*, p. 722).

values of the wave-lengths of the energy maximum we refer to Paschen's memoir already cited. The normal energy curves whose maxima are here given have the form required by formula (1) within the limits of errors, if they are treated with the proper corrections.

By formula (1) the value of c_2 should be five times the product $\lambda_m \times T$, hence $c_2 = 2890 \times 5 = 14450$ [$\mu \times$ abs. temp.]. This value also is not to be regarded as definitive.

We therefore hold that the utility of the photometric method is demonstrated, and we believe that it will furnish a very accurate value of the exponential constant, if, first, a more powerful photometer is employed which permits lower temperatures to be brought into the range of measurement, and, second, if the black body and the determination of its temperature be more perfectly realized.

TECHNISCHE HOCHSCHULE, HANNOVER.

ON THE VISIBLE SPECTRUM OF NOVA SAGITTARII

By W. W. CAMPBELL.

THE new star in Sagittarius, recently announced by the Harvard College Observatory, was observed by Mr. Wright and myself on the morning of Wednesday, April 5. The instruments employed were the 36-inch equatorial and the large Brashear visual spectroscope. A dense 60° flint prism, and eyepiece magnifying 13 diameters were used.

The star was of the 11–12 magnitude. In addition to a faint continuous spectrum, extending from $\lambda 4500$ to $\lambda 5700$, nine bright lines were observed. Seven of these lines were located by means of a fixed micrometer wire, and readings of the graduated circle, using the spectra of hydrogen and sodium for reference.

The wave-lengths and estimated relative intensities of the nine lines observed by me are given in the following table. For purposes of comparison, they are followed by the wave-lengths and estimated intensities of the principal bright lines observed in the August 1892 spectrum of Nova Aurigae, and by the wave-lengths of corresponding lines in well-known nebulae. The positions of the first and eighth lines were estimated, but not measured.

Nova Sagittarii		Nova Aurigae		Nebulae
λ	I	λ	I	λ
436?	1	436	8	436
462	4	463	7	464
470	4	468	4	469
486	10	486	10	486
496	20	495	30	496
501	60	500	100	501
518	3			518
527?	2	[527]	3	527
576	7	575	10	575

There is no doubt that these lines in the two new stars are identical. The apparent discrepancies may safely be attributed to uncertainties of measurement arising from the faintness of the spectra. The spectrum of Nova Sagittarii is therefore the spectrum of a planetary nebula.

The lines $\lambda\lambda 575$ and 436 in the new star in Auriga gradually diminished in brightness, and practically disappeared from view. The first of these lines exists in the new star in Sagittarius, and the second has a possible correspondence in the faint line near $\lambda 436$.

LICK OBSERVATORY,
April 11, 1899.

THE VARIABLE VELOCITY OF ι PEGASI IN THE LINE OF SIGHT.

By W. W. CAMPBELL.

THE velocity of ι Pegasi ($\alpha = 22^h\ 02^m$, $\delta = 24°\ 51'$) in the line of sight is variable. Partial reductions of four plates obtained with the Mills spectrograph yield the following velocities with reference to the solar system:

1897	October 7	—51 km
1898	August 19	—45
	" 29	—37
	September 28	—22

The complete reductions of the plates may change these results very slightly.

LICK OBSERVATORY,
April 10, 1899.

THE VARIABLE VELOCITY OF θ DRACONIS IN THE LINE OF SIGHT.

By W. W. Campbell.

The velocity of θ Draconis ($\alpha = 16^h\ 0^m$, $\delta = + 58^\circ\ 50'$) in the line of sight is variable. Four photographs obtained with the Mills spectrograph give approximate velocities with reference to the solar system as below.

1898	March 23 - - - - - -	+16km
1898	April 6 - - - - - -	−34
1899	April 8 - - - - - -	+10
1899	April 10 - - - - - -	−16

The complete reductions may change these results slightly.

Lick Observatory,
April 11, 1899.

A COMPARISON OF THE VISUAL HYDROGEN SPECTRA OF THE ORION NEBULA AND OF A GEISSLER TUBE.

By W. W. Campbell.

The question as to whether the spectrum of hydrogen in the nebulæ is identical with that obtained from the hydrogen in a Geissler tube is one of considerable interest. It has occupied the attention of several able observers. So far as I am aware, all the methods employed were indirect: the two spectra being observed at different times, and possibly with some of the conditions not strictly comparable.

It occurred to me that it would be a simple matter to observe both spectra simultaneously: the light from the nebula—the Orion Nebula for example—entering one half the slit, the artificial light entering the other half; so that the two spectra would be seen side by side in the eyepiece. By changing the distance of the hydrogen tube from the slit, the artificial spectrum could be reduced to any desired intensity, in order to equalize the lines in the two spectra.[1]

It was hoped that observations made in this manner would settle the question as to the relative intensities of the visual hydrogen lines in the two spectra; and, further, that the brightness of the nebular lines could be described,—from the distance between tube and slit, and from the constants of the telescope and electrical apparatus,—so that astrophysical investigators in laboratories would be able roughly to simulate the nebular lines in their natural brightness.

Comparisons of the spectra of the Orion Nebula and of a hydrogen tube were made in December, by Professor Keeler,

[1] I take pleasure in saying that, on mentioning the proposed observations to Mr. Wright, he at once outlined a similar method of direct comparison which had previously occurred to him, differing from my method only in that he proposed to use Nicol prisms for reducing the artificial spectrum to any desired intensity.

Mr. Wright, and myself. I had arranged the auxiliary apparatus so that the tube could not be moved more than two meters from the slit. The nebular spectrum being relatively much fainter than was anticipated, it was necessary to occult the entire tube, except about 2 mm of the capillary. Under these conditions the intensity of the artificial spectrum was not under good control. Nevertheless, the observations were very decisive as to the relative intensities of the lines in the two spectra; but since these observations are in accord with a second set made on April 4, under essentially perfect conditions, it will suffice to describe the apparatus used on the latter date, and the results obtained therewith.

The observations were made with the large Brashear spectroscope attached to the 36-inch telescope. A 60° flint-glass prism and an eyepiece magnifying 13 diameters were used. The spectroscope was carefully adjusted, with the slit — 1 mm wide and 13 mm long — in the $H\beta$ focus of the 36-inch object-glass. One half of the slit was covered by a total-reflection prism, whose incident and emergent faces were about 11 mm square. It was cemented in position very carefully, so that its edges were parallel to the slit, and its emergent face was parallel to the slit-plate. A light wooden rod extended to a distance of 4 meters from the spectroscope. This rod supported the hydrogen tube so that, as the telescope moved, the center of the tube remained in the normal drawn from the incident face of the prism. The tube was completely enclosed in a small wooden box, whose inner surface was painted a dull black, to avoid reflections. The long supporting rod was purposely fixed at a considerable distance to one side of the line joining the tube and prism, so that light reflected from the surface of the rod would not enter the collimator to an appreciable extent; but the additional precaution of painting the exposed surface of the rod a dull black was taken. An aperture controlled by a slide was arranged in the side of the small box toward the slit, so that any length of the capillary of the tube, up to 50 mm, could be exposed.

It is evident that, according to the above arrangement of the apparatus, the collimator lens would be filled with light from

a nebula, but that only a small portion of it would receive light from the tube. Special care was taken, in cementing the prism, and in supporting the tube, that the artificial beam of light should pass about centrally through the collimator lens, and therefore, about centrally through the prism. The question of resolving power cannot be said to have entered appreciably, on account of the very wide slit used; but the precaution was taken to mount the tube with its length parallel to the collimator, since this is the position of the tube permitting the maximum resolving power.

Inasmuch as the $H\beta$ line in the nebulæ is accompanied by the two principal nebular lines, whereas the $H\beta$ line from the tube stands alone, a diaphragm was arranged to occult the region on the red side of λ 4900 in both spectra, in order to eliminate very evident subjective effects.

The slit was directed upon the bright region of the Orion Nebula just preceding the Trapezium; and the hydrogen tube was placed 3.8 meters from the slit. The observer instructed an assistant at the tube to increase or decrease the length of the capillary exposed, until the $H\beta$ lines in the two spectra were estimated to be equally bright. The length of the exposed part of the capillary was then measured. In the same manner, the $H\gamma$ lines were equalized. Likewise, the length of exposed capillary was obtained when the $H\alpha$ line from the tube was just visible. The observations made by Professor Keeler, Mr. Wright, and myself, are tabulated below:

LENGTH OF CAPILLARY EXPOSED.

Observer	$H\beta$ lines equal	$H\gamma$ lines equal	Artificial $H\alpha$ visible
Keeler	9.0 mm	13.1 mm	7.4 mm
Campbell	6.8	12.7	6.7
Wright	7.4	13.1	15.9

The conclusions to be drawn from this table are evident.

I. When the $H\beta$ lines were equally bright, the nebular $H\gamma$ was much stronger than the tube $H\gamma$; and, for two of the

observers, the $H\gamma$ from the tube was visible, though the $H\alpha$ nebular line was invisible.

II. Increasing the length of exposed capillary until the two $H\gamma$ lines were equalized, the $H\beta$ from the tube was much stronger than the nebular $H\beta$; and the $H\alpha$ from the tube was very easily visible, whereas the nebular $H\alpha$ was invisible, as before.

III. The relative intensities of the hydrogen lines from the nebula and from the tube are, therefore, not the same: the nebular lines are relatively the stronger toward the violet, the lines from the tube are relatively the stronger toward the red end of the spectrum.

Before reaching the slit, the nebular light had passed through the Earth's atmosphere at zenith distance 60°, or through two units of thickness, and through the 36-inch lenses. Similarly, the artificial light passed through the glass wall of the tube, and through the diagonal prism. Recalling that $H\gamma$ light is more strongly resisted in its passage through such media than are $H\beta$ and $H\alpha$ light, it is clear that the real differences of relative intensity in the two spectra must be even greater than the observed differences.

If the spectra of two hydrogen sources, one of very low and the other of very high temperature, could be observed simultaneously by methods somewhat analogous to the above, the effect of temperature upon the relative intensities of the lines might become perceptible; in which case our observations would probably afford valuable evidence as to the temperature of the Orion Nebula. But this would seem to be a problem for the laboratory, rather than for the observatory.

Additional constants for defining, approximately, the brightness of the nebular lines are as follows:

The 36-inch telescope,

Aperture : focal length : : 1 : 19.3.

Electromotive force, 6 volts; Ruhmkorff coil, 20 cm in length, 10 cm in diameter, spring interrupter, 20 breaks per second, yielding a spark 22 mm long in air; hydrogen tube, by Müller of

Bonn, 24 cm long, with large capillary 6 cm long. In the December observations, 10 volts were used, with a different tube, giving the same qualitative results.

The $H\beta$ light of the Orion Nebula, in the image formed by the 36-inch telescope, is matched by the $H\beta$ component of the light radiated from 8 mm of a weakly-agitated tube, at a distance of 3.8 meters. Measured absolutely, the three principal lines in the Orion Nebula spectrum are extremely faint. They are so faint that I am wholly unable to distinguish differences in their color. In this case, and in other similar cases, of lines so faint that the observer cannot distinguish differences in their color, I doubt whether the Purkinje effect can enter to any extent whatsoever. And in all investigations of this effect, by means of measures on artificial light, it would seem to be essential that the initial intensities of the artificial lines should equal those of the nebular lines in question.

Lick Observatory,
April 17, 1899.

PHOTOGRAPHIC EXTINCTION.[1]

By EGON V. OPPOLZER.

SO FAR as known to me, Schaeberle's memoir[2] on "Terrestrial Atmospheric Absorption of the Photographic Rays of Light" is to be considered as the most extensive investigation of the effect of atmospheric extinction on the photographic determination of star magnitudes that has appeared up to the present. The discussion of his series of observations leads him to surprisingly strong extinctions at small zenith distances, which are quite contradictory to general experience.[3] He represents his observations by the purely empirical interpolation formula

$$B = B_0\left[1 - f\tan\left(\left(\frac{z}{12}\right)^2\right)\right]^2.$$

B and B_0 are the photographic magnitudes at zenith distance z and at the zenith, and f a constant depending solely on the atmospheric conditions prevailing at the time. Nothing more could be desired in the way of complexity, as the form of the expression at once shows. Indeed, if we share with Schaeberle the view—and one meets with this repeatedly from other sides—that Laplace's theory of extinction has also an empirical character, we should be justified in fitting an expression of any desired form as closely as possible to the observations. This view, however, cannot be opposed forcibly enough. Laplace's theory is built up on a perfectly natural foundation, and neglects only what is entirely permissible, in view of the slight accuracy of photometric measurements. It is therefore not surprising that the most recent observations at mountain stations satisfy the

[1] *Sitzungsberichte der K. Akademie der Wiss. in Wien.* CVII, *Abth.* II, December 1898.

[2] *Contributions from the Lick Observatory*, No. 3. Sacramento, 1893.

[3] SCHEINER, *Die Photographie der Gestirne*, p. 233, 1897.

theory in the most complete manner.[1] I desire here to bring out the foundations of the theory, because the justification of the following conclusions is thus more clearly expressed.

If a light-ray of definite wave-length and intensity i penetrates a medium, whose coefficient of absorption ν for the given wave-length as well as its refracting power a is a function of the position, the intensity J at a definite point may be found from the known law of absorption

$$J = ie^{-\int \nu ds},$$

the integral being extended along the ray to the given point. As usual ds designates the arc differential of the light-ray. Laplace now assumes that the coeffic'ent of absorption is proportional to the density or the refracting power—a law highly plausible for gases. Since the decrease of the density of the Earth's atmosphere with the height is very closely known, the integral is completely defined, and, if an isothermal decrease of density is assumed (which, as already said, is permissible in view of the inaccuracy of photometric measures),[2] we obtain the expression

$$\log \frac{J}{J_0} = -\nu_0 a_0 \left(1 - \frac{R}{a_0 \sin z}\right),$$

where

J = intensity at the observed zenith-distance z,

J_0 = intensity at the zenith,

ν_0 = coefficient of absorption for unit density of air,

a_0 = constant of refraction = refractive power at the place of observation,

R = astronomical refraction at zenith distance z.

If the constant $\nu_0 a_0$ is given for a locality of observation, the value of the extinction $\log \frac{J}{J_0}$ is deducible. ν_0 and a_0 are

[1] MÜLLER, "Photometrische und spectroskopische Beobachtungen, angestellt auf dem Gipfel des Säntis." *Potsdamer Publicationen*, **8**, 1891. "Untersuchungen über die Absorption des Sternenlichtes in der Erdatmosphäre, angestellt auf dem Aetna und in Catania. *Ibid.*, **11**, 1898.

[2] SEELIGER, *Ueber die Extinction des Lichtes in der Atmosphäre. Sitzber. München*, **21**, 247, 1891.

functions of the wave-length, and a_0 is moreover also dependent on the density, and hence on the atmospheric pressure and the temperature. On the other hand, as indicated by the theory of refraction, the factor $\left(1-\frac{R}{a_0 \sin z}\right)$ is to be regarded from the standpoint of the theory of extinction as independent of a_0 as far as $z=85°$. The values of extinction for other wave-lengths are accordingly proportional to each other, whence

$$\log \frac{J'}{J'_0} = \frac{\nu'_0\, a'_0}{\nu_0\, a_0} \log \frac{J}{J_0}.$$

The above Laplace expression holds for every wave-length, as follows from the mode of derivation, and presents itself as a matter of course when anyone undertakes the problem of photographic extinction.

Entirely aside from the confirmation of Laplace's theory, a re-discussion of Schaeberle's observations seems very desirable on account of their surprising results, which, as we shall later see, have their cause in the unfortunate choice of the expression of the function, as this expression appears entirely unsuited to represent the significant features of the extinction.

The photographic stellar magnitude is determined by the size of the star disk. The problem of photographic extinction may therefore be regarded as solved *when we can find the diameter D_0 at the zenith from the diameter D_z of a star photographed at the zenith distance z.* In order to speak at all of a photographic magnitude m', the establishment of a law of diameters

$$m = f(D)$$

is necessary, which gives a relation between the visual stellar magnitude and the measured diameter D. If this relation has been obtained for a given plate from several stars of known magnitude, the diameters of the remaining stars will conversely yield magnitudes which may now be termed photographic. In the determination of this relation it is of course necessary that the stars of known magnitude belong to a single spectral class and therefore possess the same energy-spectrum. If we provi-

sionally assume that our atmosphere is a "gray"[1] medium, *i. e.*, that it weakens each wave-length in the same ratio, the energy-spectrum of the stars will remain unchanged, and the visual magnitudes m_z altered at different zenith-distances by the extinction must be found equal to the photographic m'_z. Mathematically expressed, these two equations must hold good:

$$m_z = m'_z = f(D_z)$$
$$m_0 = m'_0 = f(D_0)$$

The subscript zero indicates that the letters refer to the zenith. These two equations give

$$\Delta m_z = m_z - m_0 = f(D_z) - f(D_0) = m'_z - m'_0 = \Delta' m_z. \qquad (1)$$

With this the problem of photographic extinction is solved, on the assumptions already made. Δm_z is the zenith reduction in magnitudes, which by the definition of star magnitudes stands in a simple relation with the above mentioned zenith reduction in logarithms of brightness, and can therefore be calculated on the basis of Laplace's theory or taken directly from the table of visual extinction. These relations exist:

$$\Delta m_z = m_z - m_0 = -\frac{1}{0.4} \log \frac{J}{J_0} = \frac{1}{0.4} \nu_0 a_0 \left(1 - \frac{R}{a_0 \sin z}\right).$$

The assumption that our atmosphere is a "gray" medium does not however fit the case. Direct observation of the setting or low Sun shows that the atmosphere extinguishes the more refrangible rays the more strongly, and this has been numerically demonstrated in an incontrovertible manner by the researches of Müller[2] and of Langley. While 10 per cent. of the red rays are absorbed, the absorption of the most refrangible rays rises up to 40 per cent. As the latter chiefly affect the photographic plate, it is evident that our assumption does not fit. Therefore the atmosphere seriously changes the energy spectrum, and hence the visual and photographic magnitudes, varied by the extinction, will deviate from each other. We shall now take this effect into account.

[1] HELMHOLTZ, *Handbuch der physiologischen Optik*, p. 280, 1867.

[2] *Die Photometrie der Gestirne*, p. 140, 1897.

The assumption is permissible that light of a limited spectral region is chiefly effective on the plate. The same thing is presupposed in visual extinction, the justification of which has been shown by Seeliger (*loc. cit.*, p. 252). This range of wave-lengths may lie for a given kind of plates at a point in the spectrum for which the coefficient of absorption is ν'_0 and the refractive power α'_0. Then we have the proportionality before mentioned

$$\log \frac{J'}{J'_0} = \frac{\nu'_0\, \alpha'_0}{\nu_0\, \alpha_0} \log \frac{J}{J_0}.$$

Passing to magnitudes and placing

$$\kappa = \frac{\nu'_0\, \alpha'_0}{\nu_0\, \alpha_0},$$

a quantity appropriately called *the constant of photographic extinction*, we get

$$\Delta m'_z = \kappa\, \Delta m_z,$$

that is, the photographic reductions to zenith are proportional to the visual. It is to be remarked here that the constant κ depends quite entirely upon the kind of plates, so that very discrepant values would be obtained for orthochromatic plates. There can therefore be no thought of an absolute constant of photographic extinction, which should depend, for instance, solely upon the constitution of the air. It is doubtless to be assumed that κ would be very nearly the same for the kinds of plates in general use.

It may be further assumed that this constant also depends on the length of exposure, since with an increase of exposure the less refrangible parts of the spectrum also come into activity, so that the effective range of wave-lengths is extended and shifted. This consideration also leads to the view that our principal assumption that a limited range of wave-lengths is chiefly effective on the plate is probably very nearly fulfilled with the comparatively short exposures, amounting at most to 16 seconds, with which we have to do. The relation last found can now be introduced in equation (1), $\kappa \Delta m_z$ being placed for $\Delta m'_z$, and we get

$$f(D_z) = f(D_0) + \kappa\, \Delta m_z.$$

This equation has now the duty of representing the photographic extinction by a new constant κ and the known visual zenith reduction. If the observations should give a value of κ near to unity, the atmosphere would be very nearly a gray medium.

I now pass to the discussion of Schaeberle's observations on the basis of the equation just obtained. We must previously decide, however, as to the function f, — as to the law of diameters. For this I shall employ Scheiner's law (*loc. cit.*, p. 215) which at once commends itself on account of its simplicity, and has stood the test for intervals up to four magnitudes. The observations to be discussed are far from reaching these limits, since with the longest exposures of 16 seconds the differences due to extinction amount at most to two magnitudes, because we do not wish to go below 80° of zenith distance. *It seems to me to be entirely wrong to pass this limit in the investigation of the photographic extinction, so long as the photographic magnitudes are determined from diameters.* The unsteadiness of the air increases considerably at these zenith distances, and is known to affect determinations of magnitude very seriously, so that it cannot *per se* be denied, that with sufficient exposure and sufficiently bright stars, the photographic diameter finally at large zenith distances increases with the zenith distance as a result of the unsteadiness of the air. To this is added that the assumptions we have made are open to suspicion beyond this limit of zenith distance, and the further circumstance that at such low altitudes small disturbances have an exaggerated effect.

By introducing Scheiner's law into the general formula we at once get ;

$$D_z = D_0 - \frac{\kappa}{b} \Delta m_z .$$

This expression could be now employed, since Schaeberle photographed the same star at various zenith-distances, on a single plate and night, with exposures of 2^s, 4^s, 8^s, and 16^s, at each zenith-distance, and then most carefully carried out measures of the diameter of the image D_z The zenith reduction Δm_z is,

moreover, calculable from the coefficient of extinction determined for other localities and from the ordinary refraction tables. Experience has shown this theoretical transfer of the coefficient of extinction to be very dubious, as it seems to be very dependent upon the place of observation. I have therefore allowed myself the very simple procedure of taking directly from Müller's table the extinction (Δm_z) as deduced from his observations on the Säntis. Since the extinctions are proportional to the atmospheric pressure, as far as we shall go in zenith-distances, it follows that,

$$\Delta m_z = \lambda(\Delta m_z) \ , \quad \lambda = \frac{658}{569} = 1.16,$$

where λ is the ratio of the pressure at Mt. Hamilton (1283 m) to that at the Säntis (2500m). The introduction of this constant of course produces no complications as it will be determined at the same time with the quantity $\frac{\kappa}{b}$, if we employ the Säntis extinctions. Our equation therefore reads:

$$D_z = D_0 - \frac{\kappa}{b} \lambda \Delta m_z .$$

Letting $x = D_0$ and $y = \frac{\kappa}{b}\lambda$, the equations of condition are

$$D_z = x - y \Delta m_z ,$$

in which Δm_z was taken directly from Müller's table. I have selected from Schaeberle's plates two belonging to the fourth series of observations to which he assigns the most weight. We shall see that the extinctions derived from these two plates by Schaeberle's formula agree with those deduced from the combination of all the series, so that these plates perfectly satisfy his conditions. Moreover, the first plate satisfies our assumptions, as it goes to zenith-distance of 80°, and shows numerous exposures at lower distances. The second plate does not satisfy our assumptions, as it contains almost exclusively zenith-distances from 80° to 89.5°; but I nevertheless include it because the comparison of the results of the plates will give the confir-

mation of what we have previously discussed. It will therefore serve only for purposes of orientation.

I wish to cite the measures of Schaeberle in full, and have collected them, with the results of the computations, in Tables I and II.

Column 1 contains the apparent zenith-distance z; column 2 the diameter D_z, in units of the fourth decimal of the English inch, which was unfortunately measured but twice by Schaeberle; column 3 the mean of the two measures; column 4 the differences (computation—observation) according to my formula from x and y, determined by the method of least squares; column 5 gives the difference ($C.—O.$) according to Schaeberle. These five columns are for the exposure of 2^s,—the others require no further explanation. The meaning of the fifth column, headed Sch., requires explanation. Schaeberle's equations of condition are

$$Q = \alpha - \beta \tan \left[\frac{z}{12}\right]^2,$$

where Q and α signify brightness. He converts the measured diameters into brightness with the aid of a table intended to exhibit the relation of star magnitude, brightness, diameter, and exposure, and the brightness first appears in the equations of condition, evidently an unnecessary introduction of new hypotheses. From the α and β found by Schaeberle by the method of least squares, I have therefore computed backward the diameters, with the aid of his table, and have compared them with the observed values, the result of which is given in column 5.

Table I shows an extremely good agreement both for Opp. and Sch., as is indeed not surprising when we examine the individual measures of the same diameter, which exhibit differences of 10 units frequently, and not seldom of 15 units. But while there is no progression in case of Opp., it cannot be denied that such occurs in case of Sch. The sums of the squares of the errors $[nn]$ are also smaller for Opp. The effect of the length of exposure is clearly shown in x and y—in x, because it denotes the diameter for the zenith, in y, because it includes the constant b, which is very dependent upon the exposure.

TABLE I.

α LYRAE.

Plate I. (Mt. Hamilton, November 4, 1891.) Barom. 658.4 mm. Therm. 13.9° C.

Z	2^s				4^s				8^s				16^s			
	Dz	Mean	Opp.	Sch.	*Dz*	Mean	Opp.	Sch.	*Dz*	Mean	Opp.	Sch.	*Dz*	Mean	Opp.	Sch.
37.2	125 125	125	+2	−1	160 145	152	+1	−3	195 195	195	+1	−2	230 220	225	−1	−6
42.9	125 120	122	−0	−2	155 145	150	0	−2	195 190	192	−1	−1	235 225	230	+6	+2
53.8	125 115	120	+1	+3	150 140	145	0	+3	190 180	185	−2	+3	225 210	217	−1	+2
64.5	120 105	112	−1	+2	140 130	135	−1	+5	185 180	182	+4	+4	210 200	205	−2	+7
73.0	110 95	102	+0	+2	130 115	122	+2	+8	170 150	160	−1	+7	185 180	182	−5	+3
77.5	95 80	87	−4	−3	100 90	95	−8	−6	145 130	137	−6	−5	170 160	165	−1	−2
79.5	90 85	87	+4	0	100 90	95	+4	−1	145 125	135	+5	−2	160 150	155	+4	−6
		[*nn*]	38	31		[*nn*]	86	148		[*nn*]	84	108		[*nn*]	84	142
	$x = 126.1$				$x = 155.5$				$x = 198.3$				$x = 231.0$			
	$y = 71.4$				$y = 107.2$				$y = 113.0$				$y = 133.1$			

Passing to Table II, the conditions are entirely reversed: although the errors for both Opp. and Sch. are within permissible limits, the agreement for Opp. is distinctly worse than for Sch., and the sums [*nn*] are greater for Opp. A decided progression is perceptible for Opp. and also for Sch., and in the same sense as for the first plate. That this poor agreement has its chief cause in the unsteadiness of the air appears from the following considerations:

Even on Plate I the decrease of diameter from zenith distance 77.5° to 79.5°, or two full degrees, is zero for the exposures 2^s and 4^s, and is extremely slight for 8^s and 16^s. The same thing appears on Plate II, for we notice that at the large

TABLE II.

α LYRAE.

Plate II. (Mt. Hamilton, November 6, 1891.) Barom. 657 mm. Therm. 8.3° C.

Z	2^s				4^s				8^s				16^s			
	D_z	Mean	Opp.	Sch.	D_z	Mean	Opp.	Sch.	D_z	Mean	Opp.	Sch.	D_z	Mean	Opp.	Sch.
48.5°	145 140	142	+6	−2	180 170	175	+14	−1	200 195	197	+13	−4	235	235	+15	− 9
59.6	140 135	137	+3	−1	165 160	162	+ 4	−1	190 175	182	+ 2	−2	220	220	+ 5	+ 1
70.5	135 120	127	−3	+2	150 145	147	− 4	+4	175 160	167	− 6	+4	210 195	202	− 4	+ 9
79.7	115 105	110	−8	+2	125 120	122	−12	+3	150 140	145	−10	+9	175 160	167	−15	+11
85.2	100 85	92	−1	+3	95 85	90	−10	+2	125 110	117	− 2	+9	125 120	122	−11	+ 2
86.1	95 75	85	−0	+3	90 75	82	− 7	−2	110 90	100	− 7	−3	115 100	107	−10	− 6
87.2	85 65	75	+4	0	80 70	75	+ 7	0	100 80	90	+ 5	−4	100 90	95	+ 7	− 7
87.5	75 60	67	0	−9	75 65	70	+ 8	−3	90 75	82	+ 4	−9	95 85	90	+12	−10

2^s	4^s	8^s	16^s
$[nn]$ 135 112	$[nn]$ 63 44	$[nn]$ 403 304	$[nn]$ 905 473
$x = 138.8$	$x = 164.5$	$x = 187.4$	$x = 224.5$
$y = 34.6$	$y = 49.3$	$y = 52.5$	$y = 69.9$

zenith-distances there occurs no rapid increase such as demanded by that extinction, but that from 85.2° to 86.1° and from 86.1° to 87.2°, or from degree to degree, the diameters decrease by the same amount. We also perceive what I have already mentioned, that for the short exposure of 2^s the unsteadiness of the air is not of the same consequence, and the best agreement occurs here. This would not *per se* be expected, as the exposure was made by raising and lowering an objective cap. An error of a few tenths of a second, unavoidable with this method of exposing, of course affects the short exposures very seriously.

In spite of the poorer agreement for Opp., to be ascribed solely to the unsteadiness, the advantage must be acknowledged to lie with my curve. For if we compare the following diameters at zenith from Plates I and II, which ought to be of the same size, under equal atmospheric conditions, identity of kind and treatment of plate (to which special attention was given), we find that my representation (Opp.) and Schaeberle's (Sch.) give these differences of diameter D^{I}_{0} and D^{II}_{0} for the first and second plate:

	2^s	4^s	8^s	16^s
$D^{\mathrm{I}}_{0} - D^{\mathrm{II}}_{0}$ (Opp.)	—13	— 9	+11	+ 6
$D^{\mathrm{I}}_{0} - D^{\mathrm{II}}_{0}$ (Sch.)	—25	—33	—17	—36

Schaeberle's differences of diameters were again obtained by calculation from his computed zenith brightnesses. While my diameters agree well on the two evenings, his do not. To get an idea of the significance of these differences, I have transformed them into magnitudes by Schaeberle's table, and obtain:

	2^s	4^s	8^s	16^s
Δm_0 (Opp.)	—0.59	—0.25	+0.21	+0.10
Δm_0 (Sch.)	—0.97	—0.75	—0.29	—0.42

These large differences led Schaeberle to exclude Plate II with the remark that that evening must have been of unusual clearness. We get just the opposite, for the exposures of 8^s and 16^s must have the determining significance, and it is not necessary for us to adopt that procedure.

Unfortunately, the plates do not enable us to determine the constant b with rigor, for then we could evaluate the constant κ. Only one way remains, viz., to determine the magnitudes from the measured diameters with the help of Schaeberle's tables of conversion, assuming their accuracy. Thus we get a table of extinctions good for the plate of November 4, on the basis of my formula, now directly comparable with Schaeberle's extinctions, since in both cases the same observations and conversion-tables were employed. This table of extinctions is calculated in Table III for the exposures 8^s and 16^s, the conversion tables for the shorter exposures being useless, as Schaeberle remarks.

Column 1 contains the zenith-distance z; 2, the diameter D_z calculated from x and y; column 3, (d_0), the diameter D_z diminished by 27 units (a correction given by Schaeberle); column 4, the photographic star magnitudes m'_z from the conversion-tables with argument d_0; column 5, the photographic zenith reductions in magnitudes deduced from the last (Opp.); column 6, the extinction for the particular plate following from Schaeberle's formula (Sch.$_1$); column 7, the extinction derived by Schaeberle from all the data (Sch.$_2$); column 8, the visual zenith reductions Δm_z for Mt. Hamilton obtained by multiplying the Säntis values by $\lambda = 1.16$.

TABLE III.

EXPOSURE 8^s.

z	D_z	d_0	m'_z	$\Delta m'$			Δm_z
				Opp.	Sch.$_1$	Sch.$_2$	
0°	198	171	—0.60	0.00	0.00	0.00	0.00
10	198	171	—0.60	0.00	0.02	0.01	0 00
20	197	170	—0.58	0.02	0.06	0.06	0.01
30	196	169	—0.57	0.03	0.14	0.15	0.02
40	194	167	—0.53	0.07	0.27	0.27	0.05
50	189	162	—0.43	0.17	0.43	0.45	0.09
60	182	155	—0.28	0.32	0.68	0.71	0.16
65	177	150	—0.17	0.43	0.87	0.89	0.22
70	169	142	+0.03	0.63	1.11	1.12	0.30
75	154	127	+0.44	1.04	1.38	1.45	0.45
80	127	100	+1.24	1.84	1.86	1.93	0.73

EXPOSURE 16^s.

z	D_z	d_0	m'_z	Opp.	Sch.$_1$	Sch.$_2$	Δm_z
0°	231	204	—0.60	0.00	0.00	0.00	0.00
10	231	204	—0.60	0.00	0.02	0.01	0.00
20	230	203	—0.59	0.01	0.06	0.06	0.01
30	228	201	—0.56	0.04	0.14	0.15	0.02
40	226	199	—0.53	0.07	0.25	0.27	0.05
50	221	194	—0.45	0.15	0.43	0.45	0.09
60	212	185	—0.30	0.30	0.62	0.71	0.16
65	206	179	—0.20	0.40	0.79	0.89	0.22
70	197	170	—0.04	0.56	1.00	1.12	0.30
75	179	152	+0.32	0.92	1.42	1.45	0.45
80	147	120	+1.16	1.76	1.65	1.93	0.73

The table shows first of all that the conversion tables yield values of m'_z in close agreement for the two exposures, which testifies as to their near approach to accuracy, and also that a certain justification for the following conclusions cannot be gainsaid.

The discussion of the observations by my formula yields much weaker extinctions at small zenith distances than Schaeberle found. I have added the two columns, Sch. 1 and Sch. 2, which show that the choice of the plates is not responsible for this, and that the extinctions derived by Schaeberle for the particular plate agree with those obtained from his table of extinction. It further follows that the photographic extinctions are proportional to the visual,—as we assumed,—being nearly twice as great, or $\kappa = 2$; a result which Scheiner[1] also surmised theoretically from similar considerations, and found confirmed on one plate, which was, however, not convincing evidence.

If we assume with Müller[2] for the most probable value of the visual coefficient of transmission t for one atmosphere,

$$t = e^{-\nu_0 a_0} = 0.83,$$

we obtain for the photographic coefficient

$$t^1 = e^{-2\nu_0 a_0} = t^2 = 0.82^2 = 0.69.$$

Therefore about 20 per cent. of the visual, and 30 per cent. of the photographic rays are absorbed at the zenith.

Now Müller[3] gives in the above mentioned research the following visually determined coefficients of transmission for the wave-lengths $\mu\mu$:

$\mu\mu$	t
560	0.82
540	0.81
520	0.79
500	0.78
480	0.76
460	0.74
440	0.71

[1] *Loc. cit.*, p. 231, or *A. N.* **124**, 276, 1890.

[2] MÜLLER, *loc. cit.*, p. 138.

[3] MÜLLER, *loc. cit.*, p. 140, or *A. N.*, **103**, 241, 1882.

We may with sufficient accuracy extrapolate for the wave-length $434 \mu\mu$, which represents approximately the maximum of sensitiveness of ordinary plates,[1] and we obtain as a Müller coefficient of transmission for the most active photographic rays,

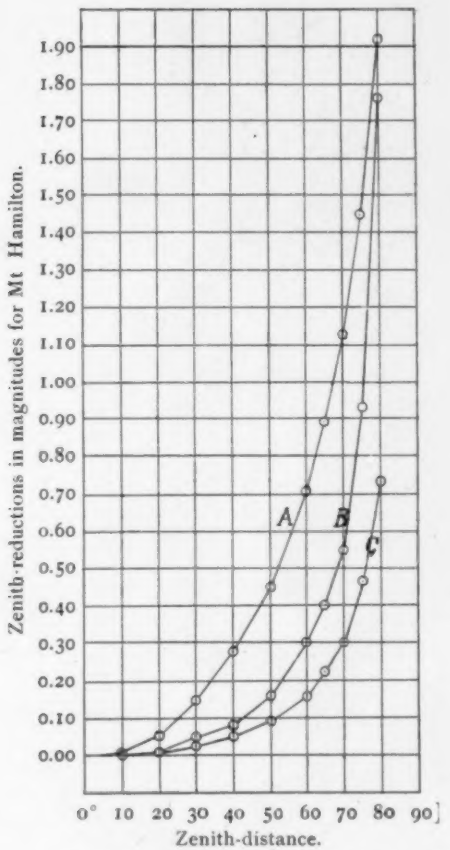
$$t'' = 0.69,$$

a value in perfect agreement with that we have derived from Schaeberle's observations, thus confirming Scheiner's surmise and all of our assumptions. The constant of photographic extinction κ is therefore determined above to be about 2, and, as already stated, it can be changed with other kinds of plates or other exposures.

It should be mentioned that Schaeberle employed Seed's No. 26 plates, 4×5-in. in size.

It is especially evident from Table III that the strong extinctions found by Schaeberle at slight zenith distances are not real, but have their cause solely in the unfortunate form of the interpolation formula. Every function which is to represent the extinction must have the property of increasing slowly at slight zenith distances ($z < 60°$), and then of suddenly increasing strongly. If Schaeberle's function is to represent the values at great zenith distances, which was its chief object, then a steep increase of the extinction curve is necessary at small zenith distances. This is not the case, and to make this more conspicuous, I have added the figure, which represents the extinction curve according to Schaeberle and myself, as well as the visual extinction, and requires no further explanation.

[1] SCHEINER, *loc. cit.*, p. 231.



A = Photographic Extinction (Schaeberle).
B = Photographic Extinction (Oppolzer).
C = Visual Extinction.

THE ABSOLUTE DETERMINATION OF THE RADIATION OF HEAT WITH THE ELECTRIC COMPENSATION PYRHELIOMETER, WITH EXAMPLES OF THE APPLICATION OF THIS INSTRUMENT.[1]

By Knut Ångström.

1. With the increasing interest attending researches in the field of radiation, the physical methods for the absolute determination of the intensity of radiation have gained a greater importance. The older forms of apparatus, which were chiefly intended for the measurement of the Sun's radiation, no longer satisfy the present requirements as to refinement and sensibility. The newer forms of apparatus, with which absolute measures can be procured, seem to have met with no very general adoption, as appears from the fact[2] that most investigators in this field are satisfied with rather arbitrary determinations of the absolute value of a radiation, and generally with relative measures. Thus, for example, the sensibility of the bolometer and thermopile is obtained from the deviation of the galvanometer needle, produced by a definite increase of temperature of the absorbing surface, or by the radiation of a poorly defined source of heat. A comparison of the results of different observers and a recalculation of the determinations in absolute units is often impossible.

As early as 1886, I brought forward a method which has proved useful, not only for the determination of the Sun's radiation, but also for performing other physical experiments in the laboratory.[3] Later, I described still another method which clearly surpasses the former as regards sensibility, and especially

[1] *Wiedemann's Annalen*, **67**, 633, 1899.

[2] In the ordinary text-books of practical physics, there is given, so far as I know, not a single method for the absolute determination of the radiation of heat.

[3] K. Ångström, "Sur une nouvelle méthode de faire des mesures absolues de la chaleur rayonnante," *Acta Upsal.*, 1886; see also *Wied. Ann.*, **39**, 294, 1890. The method has been used by me for different researches. See *Wied Ann.*, **48**, 517, 1893, or *Acta Reg. Soc. Upsal.* 1892.

in the convenience of operation.[1] Since only a preliminary notice of this method has been described, and that in periodicals which are hardly generally accessible, I will now give in this place a complete description, and at the same time relate my experience in the construction and use of the apparatus which I have acquired during six years of its use.

2. *The principle of the method is briefly as follows.*[2]—Of two metal strips, blackened on one side, and in every way similar, one is exposed to the radiation to be measured, the other, which is screened from the radiation by a double wall, is warmed by an electric current. If the strength of the current is regulated in such a manner that the warming of the two strips is the same; then the energy of radiation is equal to the energy led in by the electric current. Let q be the radiation in units of a second and square centimeter, b the width, a the power of absorption, r the resistance for unit length of the strips, and, finally, i the strength of the electric compensation current, then:

$$baq = \frac{ri^2}{4.18}.$$

From this we get

$$q = \frac{ri^2}{4.18\,ba} \text{ gram-calories per second and square cm,}$$

or

$$Q = \frac{ri^2}{4.18\,ba}\,60 \text{ gram-calories per minute and square cm.}$$

[1] K. ÅNGSTRÖM, *Acta Upsal.*, June 1893. *Physical Review*, 1, 365, 1893.

[2] F. KURLBAUM has, in a short notice in the *Berichten der Thätigkeit d. Phys-Techn. Reichsanstalt* in 1891 and 1892 (published in Nov. 1892) and in the *Zeitschrift für Instrumentenkunde*, March 1893, p. 122, suggested a similar principle for the measurement of radiation in absolute measure, which he developed further in *Wied. Ann.*, **51**, 591, 1894. He later employed his method for the determination of the radiation of a black body (*Wied. Ann.*, **65**, 746, 1898). We each, therefore, had almost simultaneously, but evidently independently, an idea in many respects identical. But as Kurlbaum himself remarks (*Wied. Ann.*, **51**, *l. c.*), we suggested entirely different methods of carrying out the idea. Without entering here upon a comparison of our two methods, I would only remark that the principle of the bolometer is not employed in my instrument, and further that my apparatus is constructed in entire symmetry, two similar strips being employed, that the equality of temperature is measured by thermo-elements, and finally that the effect of the radiation and of the electric current are simultaneously observed.

It is at once clear that by this method we need not take account of corrections for the dissipation of heat through radiation, convection, or conduction, since, from the equality of temperature of both strips, these corrections are therefore the same for both, and on that account the calculations are done away with. With this method we need therefore to determine only once for all the constants r, b, and a, and for each determination of a radiation to observe only the strength of the current i, in order to obtain the radiation in absolute measure. Since r changes slightly with the temperature, this change must be included in the calculation.

The preparation of the two metal strips must naturally be made with the greatest of care. I proceed in the following manner: A strip of platinum foil from 0.001–0.002 mm thick, and of about four sq. cm surface is laid on a glass plate. A small piece of the thinnest of silk paper, — a trifle larger than the platinum foil, is dipped in a solution of shellac and applied to the strip. The superfluous shellac solution is brushed off, and the bubbles of air between the platinum strip and the paper carefully removed. After a complete drying, the glass plate is fastened to the table of a dividing machine, and the platinum strip is cut up into pieces of proper width, which afterward can easily be separated from the glass plate.

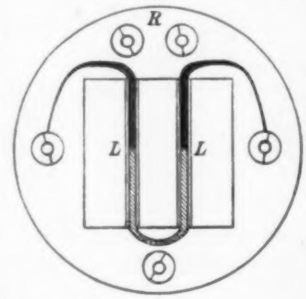


FIG. 1.

Two such strips lying next each other are tested as regards their electric resistance, and if the difference between them is not more than a few per cent., they are fastened to a small ebonite frame R (Fig. 1, in natural size). On the paper side of the strips the thermo-element is fastened by means of a little shellac, and in such a manner that the junctions L lie about at the middle points of the strips. The thermo-element consists of a U-shaped piece of a very thin sheet (about 0.02 mm thick) of "constantan" or nickel, to which is soldered a plate of cop-

per of the same thickness and width. In order to complete the symmetry for power of radiation of the back sides of the strips, these sides were coated with black varnish.

In order to blacken the strips on the front sides, they were first coated galvanically with a thin layer of zinc, and then treated with a 1 per cent. solution of platinum chloride until the resistance, which had been slightly lessened through coating with zinc, reached its former value. In order to raise the coefficient of absorption, the strips were finally given when cold a thin coating of lampblack. This coating was obtained from a stearin candle, in the flame of which was held a fine net of copper wire. The readiness of the strips for use is tested by connecting the thermo-element to a sensitive galvanometer. If the position of rest of the galvanometer does not chance for simultaneous radiation upon the two strips, the symmetry is complete.

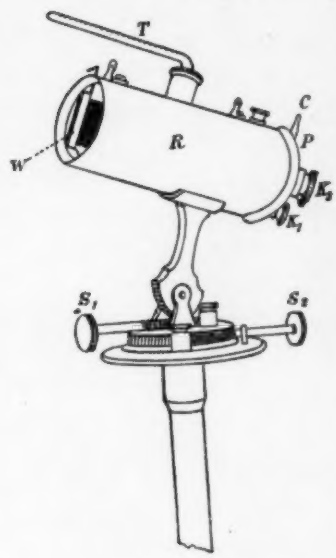


FIG. 2.

The mounting of the apparatus is shown in Fig. 2 (one third the natural size). The strips are placed in a tube R, which is provided with three diaphragms. The tube can be pointed in any desired direction by two screws S_1 and S_2. The temperature in the tube can be determined by means of a thermometer T. A small double-walled, reversible screen W, which is fastened in the front end of the tube, protects the one strip from the radiation. The back side of the tube is closed by an ebonite stopper

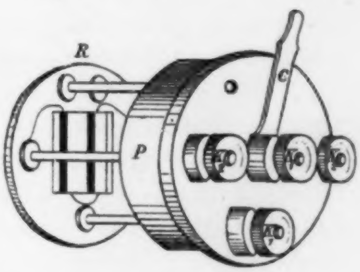


FIG. 3.

P. This, which is shown in Fig. 3 (two thirds natural size), carries the binding screw K_1 for conduction to the strips, and the binding screw K_2 for the thermo-element, as well as a small commutator *C*, in order to throw the current from the one to the other strip.

3. Concerning the *constants of the apparatus*, the width of the strips was already determined on cutting them with the dividing machine. In order to control these widths, after completing a few instruments, I again measured these widths with the dividing machine. This gave, for example, for the instrument No. 2 used below, in place of the desired width of 2 mm, the following values:

	Right strip	Left strip
	2.00 mm	1.99
	2.02	2.01
	2.01	
Mean:	2.01 mm	2.00 mm

Since the coating with lampblack leaves the edges a trifle rough, an error of 0.01 mm in the measures of the width evidently cannot be avoided, which in the width of the strips here used may make an error of 0.5 per cent. in the final value.

In order to determine the *resistance of the strips*, I have used the electrometric method, which here offers striking advantages. The arrangement of this experiment is seen in Fig. 4. From the galvanic cell *S* a current is sent through the pyrheliometer strip *F G*, and a calibrated wire of German silver or manganin of accurately known resistance. From the two knife

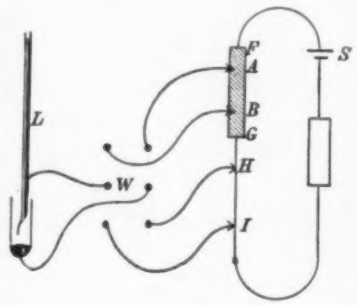


FIG. 4.

edges *A* and *B*, and from two contacts *H* and *I*, which are in connection with the strips and the wire, and of which *I* is a sliding contact, wires go to the commutator *W*, and from here to the Lippmann capillary electrometer *L*.

The arrangement of the knife edges *A* and *B* is seen in Fig. 5. These are fastened at the ends of a small glass tube *E*, which is suspended from an ebonite plate *H* by spiral wires. This plate can be lowered by means of a screw, and so the knife edges can be pressed lightly on the strips. Since the sliding contact is so regulated that the potential differences between *A* and *B* and between *H* and *I* are the same, the resistance of the strip between the points *A* and *B* is directly determined. By this method of determination, the disturbances at the ends of the strips are eliminated, and the resistance determined in the part of the strip lying close to the junction of the thermo-element.

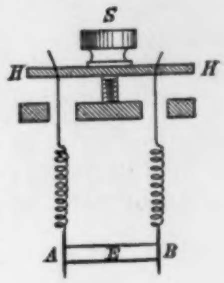


FIG. 5.

The platinum strips are heated by the passage of the current, and accordingly the resistance increased. By investigating the resistance for currents of different strengths, the relation between these two magnitudes can be approximately determined. Accordingly, for instrument II at 19°, the following results were obtained:

Strength of current		Resistance in ohms per cm	
I	I^2	Observed	Calculated
0.378	0.1428	0.322	0.3226
0.289	0.0835	0.316	0.3170
0.228	0.0520	0.314	0.3141
0.188	0.0354	0.312	0.3125
0.141	0.0199	0.3105	0.3111
0.090	0.0081	0.3100	0.3100
0.000	0.0000		0.3092

Since the heating of the strips is proportional to the square of the strength of the current, we have, if m'_i and m'_0 are the resistances for currents of strength i and o,

$$m'_i = m'_0 \, (1 + \beta i^2).$$

The values of m_i in the above table, calculated according to the above formula from the values $m_0 = 0.3092$ and $\beta = 0.303$, show with certainty that this method of measuring leads to very good results.

The temperature coefficient of the electric resistance of the strips is found in the ordinary manner by means of a Wheatstone bridge. The formula

$$m''_t = m''_0 (1 + \alpha t)$$

gives here a satisfactory degree of precision. For the platinum material used I have found $\alpha = 0.00216$; and this gives $m''_0 = 0.2970$.

Denoting by m_{it} the resistance to a current of strength i at a surrounding temperature t, then we have:

$$m_{it} = m_0 (1 + \alpha t) (1 + \beta i^2).$$

This method, which is simply to calculate the resistance from the temperature of the field, and from the strength of the current, is not quite exact, since the heating of the strips depends on the rate of cooling, and this is not constant. But since the correction to the resistance caused by the heating of the current amounts, at most, to 3 per cent.[1] and, besides, the strips are fairly well protected from currents of air, this method of correction is quite satisfactory in most cases; on account of its simplicity and convenience it can be unconditionally recommended. If we assume a change of 10 per cent. in the rate of cooling, which must be an upper limit, we make an error in the calculated resistance of 0.3 per cent., which causes an equal percentage of error in the determination of the radiation.

Of the determinations of the constants of the apparatus, the *coefficient of absorption of the strips* is the most difficult. Ordinarily, for measures of heat radiation, the absorptive power of a lamp-black surface is taken as equal to unity — although it is generally recognized that this supposition is not correct — or a rather arbitrary value of this absorptive power is taken without any special researches on the action of the surfaces. Since surfaces produced in different ways show quite a difference in their absorptive power, it is important to examine a surface of just the same character as the one used in the instrument. To this

[1] The correction is as much as this only in the determination of the strongest of the solar radiations.

end I have made an investigation of the absorptive power of the surfaces made in the manner described above, by the determination of the diffusion of different kinds of radiation.

The results of these investigations, which I have more fully discussed in another place,[1] and on that account I will only mention them here, are briefly as follows:

a. The absorptive power of the platinum surfaces increases only insignificantly on being coated with lampblack, but it becomes more uniform for different wave-lengths.

b. The platinum and lampblack surfaces have a coefficient of absorption which is slightly selective, as it increases with increasing wave-lengths.[2]

c. For solar radiation, the mean coefficient of absorption lies between 98.3–98.8 per cent. according to the thickness of the coating of lampblack.

d. If we take the coefficient of absorption of these surfaces as constant for different wave-lengths, and equal to 98.5 per cent., then we make at most an error of 0.5 per cent., in the determination of the intensity of radiation.

4. *Method of Observation.*—As I have already shown in the preliminary communication, the observations with this apparatus can be made in different ways. The same galvanometer which is used for the determination of the temperature can also be used for measuring the strength of the current. It is, however, better, and much more convenient, to use for this purpose two different instruments. Any galvanometer, or galvanoscope of rather high sensibility, can be used in connection with the thermo-element; for the determination of the strength of the heating current, I have generally used an electro-dynamometer especially constructed for it. Still the precision milliamperemeter of Weston or Siemens and Halske can be used with great advantage. Since this instrument is found in nearly every

[1] K. Ångström, Öfversigt af K. Vet. Akademiens Förhandl., Stockholm. No. 5, p. 283, 1898.

[2] I wish to emphasize here the difference between the absorptive power of lampblack and the absorptive power of a lampblack surface. The former decreases, as I have shown, with increasing wave-lengths. *Wied. Ann.*, **36**, 715, 1893.

physical laboratory, I shall assume in what follows that the measures can be made with this apparatus.

Fig. 6 shows the arrangement of the apparatus. S is the battery, a Daniell's or a Leclanché cell, or, still better, an accumulator; R and R_1 a plug and slide wire rheostat; A the milliamperemeter; C the commutator to the strips. The galvanometer G is joined to the thermo-element. For making a determination, the apparatus is first oriented in the proper direction toward the source of heat by the help of sights on the tube, the covering at the mouth of the tube is taken away, and the small screen turned in such a manner that the radiation falls on both strips, and then the position of rest of the galvanometer can be determined. The screen is then turned toward one side, and at the same time the current is closed through the shaded strip and regulated in such a manner that the galvanometer takes the same position of rest as before. After the strength of the current is read off, the screen and commutator are reversed, and then the determination repeated. In this way in a few minutes many pairs of determinations can be obtained. The thermometer projecting into the instrument tube gives the temperature of the air close to the strips.

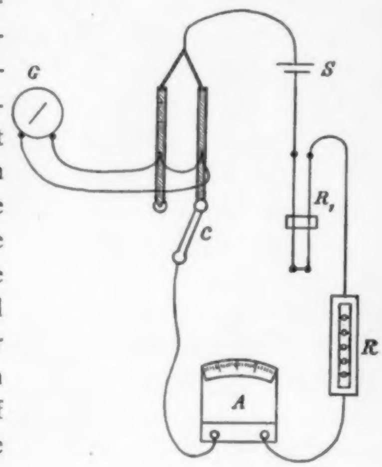


FIG. 6,

5. The errors in the resulting determination with this instrument depend first, on the accuracy with which the constants of the instrument are determined, and, second, on the accuracy with which the settings for the same temperature and the observations of the strength of the current can be carried out. Since

$$\frac{dQ}{Q} = \frac{dr}{r} - \frac{db}{b} - \frac{da}{a} + \frac{2\,di}{i},$$

we find, in agreement with the above, that the error which will be caused through *dr*, *db*, and *da*, will amount at most to 1.3 per cent. The error in *di* depends, naturally, in a great degree, on the special conditions under which the measures are taken. The strength of the current can be determined without difficulty, to within 0.3 per cent. The error in Q resulting from *di* will, therefore, at most, not exceed 0.6 per cent. The total error of a single determination of Q will, therefore, amount at most to about 2 per cent., of which about 1.3 per cent. is constant, and 0.6 per cent. is accidental error.

6. As proof of the reliability of the method, the following observations may be given, viz.: first, the determination of the radiation of a lamp with two instruments, one of which I made more than four years ago,[1] the other having been lately constructed; second, the comparison of the observations in solar radiation with the two instruments, the differential pyrheliometer and the compensation pyrheliometer.

The lamp was a so-called "focus lamp" (from the factory of "Svea" in Stockholm) of 32 candle-power. The current was generated by twenty-four large Tudor accumulators. The distance between the source of heat and the strips of the apparatus was 50.4 cm. R denotes the amount of the radiation of the right, L the radiation of the left strip.

It was found:

WITH APPARATUS II. (OLD).

	Strength of current.	
R_1	0.0412	Temperature 17.0°
L	0.0410	
R_2	0.0411	

$$\text{Mean: } I = \frac{R_1 + R_2 + 2L}{4} = 0.0411$$

Resistance = 0.308 ohms. $Q = 0.0380$ *g*-cal. per min. and sq. cm.

[1] This instrument, denoted by II, is one of those which I have used in two series on Teneriffe and one series in Switzerland.

WITH APPARATUS XI. (NEW).

	Strength of current.	
R_1 - - -	0.0390	Temperature 17.0°
L - -	0.0390	
R_2 - - -	0.0388	

$$\text{Mean: } I = \frac{R_1 + R_2 + 2L}{4} = 0.0390$$

Resistance = 0.339 ohms. $Q = 0.0376$ g–cal. per min. and sq. cm.

This example shows that the different instruments agree very well with each other, and that, if not injured, they do not change with time. I might easily confirm these results still further with other observations.

It was of special interest to me to compare this new apparatus with my older one (the differential pyrheliometer). During the summer of 1894, I had the opportunity during my stay in Borgholm, on the island of Öland, to carry out some observations for comparison. I was engaged in making some changes in an apparatus for registering the strength of the solar radiation, and accordingly measured also the intensity of this radiation with the two instruments mentioned. As I had no assistant, and had only one galvanometer with me suitable for the experiments, I could not carry out the determinations exactly simultaneously. The registering instrument enabled me to take into account the changes in the solar radiation, and to reduce the indications of both instruments to precisely the same time. The following small table contains the results of these observations:

1894		Determ. with the Comp. Pyrheliometer	Determ. with the Diff. Pyrheliometer	Diff.
10 Aug.	11:20 A. M.	1.215	1.25	−0.030
11 "	4:24 P. M.	0.995	0.99	+0.005
12 "	11:28 A. M.	1.178	1.18	−0.002
15 "	6:21 P. M.	0.659	0.64	+0.019
19 "	8:27 A. M.	1.134	1.14	−0.006
19 "	12:38 P. M.	1.359	1.37	−0.011

The good agreement between the determinations of these two instruments, so different in principle and operation, bespeaks, it seems to me, the reliability of both. It must be once more

mentioned that the constants of the new instrument are easier to determine, that the sensibility of it can be increased more easily, and, finally, that the determinations can be carried out in a much shorter time.

7. *Application to the determination of the strength of the solar radiation.*— I used the new method at first for the determination of the solar radiation, and that at different heights above sea level. A very considerable number of observations are already available for this question, but since these have been made for the most part with rather unreliable forms of apparatus and under unfavorable meteorological conditions, further observations appear to me necessary. I therefore put together an easily portable instrument such that the compensation pyrheliometer with rheostat, a small galvanoscope, an electrodynamometer specially constructed for this purpose, of a sensibility easily controlled, and a Leclanché cell, were so arranged in a small box 44 cm high, 27 cm wide, and 12 cm deep, that the instrument could be set up in a few minutes for a determination. The weight of this apparatus, with a small stand for mounting, was only seven kilograms. Teneriffe was selected as the place of observation, its uniform climate being very favorable for this purpose.

I expect soon to publish[1] the results of two series made on Teneriffe during the summers of 1895 and 1896, in which more than 600 absolute determinations were made. Here I will give only a single complete determination of the solar radiation, and two series of observations at different altitudes, in order to show how excellently well adapted the apparatus is for such measures.

As an example, I give a measurement made on the 25th of June, 1896, under very difficult circumstances on the highest point of the peak of Teneriffe.

Extract from observing record on 25th of June, 1896:

Height above the sea, 3700 m. Height of barometer, 495.1 mm. Wind: very strong, S. W. Temperature, 9.2°. Tension of water vapor, 2.44 mm.

[1] I hope to publish this during 1899 in the *Verhandl. d. Schwed. Akad.*

Time	Strength of the comp. current squared	Temperature in the tube of the instrument
	R_1 0.0704	
$11^h\ 55^m$	L 0.0713	21.0°
	R_2 0.0702	

$$\text{Mean} = \frac{R_1 + R_2 + 2\,L}{4} = 0.708$$

$$m_{ti} = 0.3164, \quad Q = 1.626$$

	R_1 0.0707	
12^h —^{m}	L 0.0699	21.5°
	R_2 0.0694	

$$\text{Mean} = \frac{R_1 + R_2 + 2\,L}{4} = 0.0700$$

$$m_{ti} = 0.3169, \quad Q = 1.614$$

As an example of the daily series, I cite the measures of July 3, 1896, on which day observations were made at the same time by me at Guimar, 360 m above the sea, and by my assistant, Hrn. Edelstam, in "Alta Vista," 3252 m above the sea.

Guimar. Barometer 731.3			Alta Vista. Barometer 518.9		
Time	Altitude of Sun	Q	Time	Altitude of Sun	Q
$5^h\ 49^m$ A. M.	8° 16′	0.721	$5^h\ 30^m$ A. M.	4° 26′	0.819
5 55	9 31	0.795	5 58	10 8	1.138
6 28	16 24	0.994	6 29	16 36	1.340
6 57	22 32	1.105	6 59	22 57	1.421
7 26	28 45	1.190	7 31	29 49	1.488
7 57	35 27	1.251	7 59	35 53	1.525
8 57	48 34	1.292	8 58	48 49	1.578
9 58	62 0	1.361	10 5	63 32	1.609
12 1 P. M.	84 27	1.384	12 3 P. M.	84 26	1.618
1 58	62 52	1.325	1 53	63 58	1.609
2 58	49 40	1.275	2 58	49 40	1.579
.. ..	.. ..	.....	3 31	42 26	1.540
3 56	36 58	1.219	3 54	37 24	1.520
4 27	30 15	1.150	4 29	29 49	1.479
4 55	24 14	1.070	4 55	24 14	1.439
5 27	17 27	0.943	5 13	20 23	1.396

The extraordinarily regular course of the series of observations at Alta Vista is a proof, not only of the beauty of the apparatus, but also of the favorable atmospheric conditions.

8. *Application of the apparatus for the determination of a Hefner normal lamp.*—For many researches on radiant heat it would, without doubt, be very advantageous to have an accessible "normal radiation" at our disposal. The radiation of an "absolute black body" at 100° would probably first suggest itself; this normal is, however, not very convenient, since it demands especially good conditions and an accurate determination of the temperature, not only of the radiating surfaces, but also of the screen used. In the hope that a Hefner normal lamp might be applicable for the total radiation, where no greater accuracy is required, I determined its radiation. From researches on the luminous radiation of this lamp, we already know that it is subject to rather large changes, and it is to be assumed *a priori* that these are still greater for the invisible radiation. In order to eliminate the radiation from the products of combustion and from the flame tube, a small double screen of "metal paper," with the polished surfaces together, was put on the lamp. In the screen was a rectangular opening 40 mm long and 14 mm wide. The strips of the compensation pyrheliometer were placed about 30 cm from the flame. I first cite a single determination:

	i	
R_1	0.0259	Temp. 18.0°
L	0.0260	
R_2	0.0242	

$$\text{Mean} = \frac{R_1 + R_2 + 2\,L}{4} = 0.0256 \qquad m_{ti} = 0.308$$

$$Q = 0.0145 \text{ g-cal. per min. and sq. cm.}$$

In this manner the following four series were made, of which I and II were made on the same day, the others about two weeks later:

Series I T=18.0°		Series II T=17.5°		Series III T=17.5°		Series IV T=16.7°	
I	Q	I	Q	I	Q	I	Q
0.0261	0.0152	0.0244	0.0134	0.0255	0.0145	0.0258	0.0149
0.0262	0.0153	0.0250	0.0140	0.0254	0.0144	0.0261	0.0152
0.0263	0.0154	0.0248	0.0138	0.0259	0.0150	0.0259	0.0150
0.0261	0.0152	0.0251	0.0141	0.0258	0.0149	0.0260	0.0151
0.0260	0.0151			0.0259	0.0150		
0.0258	0.0149						
0.0256	0.0146						
Mean	0.0151	Mean	0.0138	Mean	0.0148	Mean	0.0151

The agreement of the results of series I, III, and IV is very good; on account of some unknown cause, the result of series II differs considerably from the rest. The mean value of the result is 0.0147, the greatest deviation from the mean about 5 per cent. It therefore appears to me not impossible that, by taking special precautions, we might find in the Hefner lamp a really useful normal radiation. By the application of the law of decrease of intensity of radiation with the square of the distance from the source of radiation, the strength of the radiation of a Hefner lamp in a horizontal direction can be calculated as 132 g-cal. per min. and sq. cm, which may for the present be considered an approximate value.

I trust I have shown by this article that the electric compensation pyrheliometer is a reliable, handy, and sensitive instrument, which is as good for measures of solar radiation as for the determination of weaker sources of heat in the laboratory, and on this account is applicable for the determination of the constants of the bolometer and thermopile. The instrument was made to my perfect satisfaction by H. Sandström, mechanician in Lund.

UPSALA, January 1899.

ON THE PRESSURE IN THE SPARK.[1]

By Eduard Haschek and Heinrich Mache.

During their experiments with their chloride of silver battery of 11,000 cells, Warren de la Rue and H. W. Müller[2] observed that a stream of sparks causes a sudden rise of pressure in the partially evacuated bell-jar of an air-pump, which falls just as quickly on breaking the current. This expansion, of which an example is given of 15.8 mm at a pressure of 56 mm of mercury, cannot be regarded as a thermal expansion, on account of its large amount and its rapid disappearance, after the interruption of the current, but must be considered as a consequence of an elevation of pressure in the metallic vapors in the path of the spark, as was at the time suggested by de la Rue and Müller. We may also infer the existence of such a pressure in the spark in an indirect way, for Humphreys[3] has proven that the lines of the arc-spectrum suffer a displacement toward the red as soon as the arc is burned in a gas of increased pressure. On the basis of Humphreys' figures, we can conclude from a displacement of lines as to the elevation of pressure in the part of the luminous gas from which the light in the spectroscope comes.

In the course of their researches on the ultra-violet spark spectra of the elements, F. Exner and E. Haschek[4] have demonstrated very considerable displacements of this kind, and by comparison with Humphreys' results have estimated the spark pressure to be from 24 to 30 atmospheres. An observation by Mohler[5] is also in agreement with this, which indicates that the

[1] *Sitzungsberichte der Kais. Akad. der Wissenschaften in Wien.* **107**, Abth. IIa, November 1898.

[2] *Proc. R. S.*, **29**, 286, 1879; *Comptes Rendus*, **89**, 637, 1879.

[3] This Journal, **6**, 1897.

[4] *Wiener Sitzungsberichte*, **106**, 1897.

[5] This Journal, **4**, 1896.

same wave-lengths are obtained from a Geissler tube exhausted to 2 mm pressure as from the arc at 20 mm.

Finally it has been directly proven by H. Mache[1] that alterations of the pressure around the electrodes occur on the discharge of high tension electricity from points, as well as on the passage of sparks from an influence machine or induction coil.

It seems desirable from all this to study this interesting phenomenon more closely and quantitatively.

§ 1. We arranged the final experiments as follows: In a strong glass globe of 20.8 cm diameter ground stoppers *A* and *B* were inserted above and below. The one electrode was introduced in *A*, while at *B* a barometer tube one meter long was fused on, carrying within it a steel rod to which the second electrode was attached. The rod was connected with a steel screw of 2 mm pitch, so that the lower electrode could be raised and lowered. An air-tight joint was effected by a vaseline oil or mercury column, from the height of which the pressure in the globe could be read off. The current from a transformer of some 5200 volts effective pressure was conducted to the electrodes. In case of necessity condensers of measured capacity could be switched in parallel to the spark-gap. The globe could be exhausted or filled with different gases through two cocks. It proved to be very often necessary to remove the nitrous oxide which developed abundantly during sparking.

§ 2. If the spark is allowed to pass between the electrodes the index liquid in the barometer tube indicates a rise of pressure which falls back immediately and almost completely when the current is shut off. It is necessary for us to establish for ourselves a relation between the rise of pressure in the whole space of the globe and the rise in the spark itself, in order to interpret the former, which is directly measured. It is at once clear that this sudden rise of pressure in the spark, which disappears quite as rapidly after the discharge, will give occasion for the development of a wave which will progress with at least the velocity of sound. On account of the small size of the source — we employed

[1] *Wiener Sitzungsberichte*, **107**, 1898.

a spark of more than 3 mm in length in only a single case—these waves may be regarded as spherical even at a short distance.

Thus there will spread over the unit of surface of each such spherical wave a pressure which will decrease with the expanding radius of the wave, but nevertheless in such a manner that the pressure on the total surface remains constant, for the total energy of the wave, to which the pressure is directly proportional, remains constant. If we therefore call P the rise of pressure on the unit surface of the spark itself, whose surface is O, there will spread over the unit of surface of a spherical wave of radius r the rise of pressure:

$$P^1 = \frac{O}{4\pi r^2} P \quad .$$

If r is the radius of the glass globe employed, this pressure P^1 is observed directly at the manometer, since the wave progresses without losses of energy in the attached tube, which is sufficiently calibrated.

But since a single spark is not sufficient for measurement on account of the inertia of the index liquid, we are forced to await the setting of the index during a stream of sparks, so that P^1 and similarly P must be defined as a time mean of the elevation of pressure. It is however obvious that this time mean P can be set the more nearly equal to the true spark pressure, the quicker the separate discharges follow each other.

A number of sparks per second is experimentally found beyond which no variation in the setting of the index liquid can be distinguished. If we consider that the metallic vapor developed, and hence the pressure prevailing between the electrodes, cannot disappear absolutely at the same time with the spark, we may assume at the start that this limiting value was sufficiently reached with our transformer, which was fed by a current of about 80 alternations per second. In fact the validity of this assumption was established by the experiments with the induction coil, to be described in §9.

It only remains to describe the method by which we attempted

to determine with the closest approximation the surface O of the spark, which enters the computations. For this purpose we prepared instantaneous photographs of the spark, enlarged fourfold. The speed of the shutter employed was $\frac{1}{25}$ second. The portion of space thus cut in a plane and illuminated by the spark was measured with a polar planimeter. In our case it was always sufficient to substitute for the surface corresponding to the cross-section thus measured the surface of a sphere whose radius ρ is equal to the radius of the circle of equal surface with that section. Then

$$P^{1} = \frac{\rho^2}{r^2} P \text{ and } P = \frac{r^2}{\rho^2} P^{1}.$$

The measurement of P^{1}, the elevation of pressure in the whole space of the sphere presented no especial difficulties. There always occurs, indeed, a throw of the index liquid in consequence of the heat developed in the spark, but this is so different in its nature from the pressure throw that they can be readily distinguished. One fact should be noted here, however. During the experiment the electrodes become very considerably heated, with too strong a primary current even to red heat. The rising warm air, by its better conductivity, reduces the potential of the spark markedly, and the throw simultaneously begins to go down.

This effect also shows itself acoustically, for the loud and almost unendurable rattle of the sparks in free air passes into a weak, hissing sound. The observations may, therefore, be at the longest continued until this begins.

It should be further remarked that we hold the heat of the spark to be a phenomenon of a secondary nature, directly caused by the kinetic energy residing in the metallic particles or molecules thrown off from the electrodes by the electromotive forces. In this we find ourselves in agreement with experiments of Schuster[1], who assigns to the particles thrown off from the electrodes velocities as high as 2000 meters per second.

The view here stated has, moreover, been for a long time

[1] *Nature*, 57, 17, 1897.

repeatedly expressed. It is as far as we know first stated by v. Waltenhofer,[1] who imagines "the ponderable matter between the electrodes, not only as carrier of the discharge, but also as itself set in motion by it."

§3. We first determined by the process described above the pressure in the spark for different capacities introduced into the secondary circuit. We employed here a spark-gap of 2 mm between brass poles of 3 mm diameter. We used vaseline oil as the indicating liquid. In these experiments the primary current amounted to 7.5 amperes, with a voltage of 100.

The results are collected in the following table:

TABLE I.

Capacity in meters	Throw in mm vaseline oil	Pressure in the spark in atmospheres
5.16	33	22
11.1	118	40
16.4	148	48
22.9	165	45
53.1	220	51
77.3	228	50
100.2	258	46
156.0	293	36

The air enclosed in the sphere was at a pressure of 695 mm.

In this and in the following tables we give the direct reading at the manometer because we see in this a measure for the sound energy in the spark, according to the view developed in §2. It appears at once that with increasing capacity and hence increasing quantity of electricity which passes in individual sparks, this acoustical energy increases—at first very rapidly, but later rising toward a definite limit. At the same time the pressure in the spark also increases rapidly at first, reaches a maximum, and then slowly falls off in consequence of the enlargement of the volume of the spark.

§4. The phenomena were next more closely examined which occur when the length of the spark is varied, the capacity and primary current remaining constant. Here again, the observed

[1] *Pogg. Ann.* **128**, 608, 1866. See also, Lecher, *Wiener Sitzungsberichte*, **96**, 103, 1887.

manometer readings rose toward a limit. The pressure developed in the spark appears to remain constant, however, since the number of sparks does not change, and hence the amount of energy passing in the single spark remains unchanged. The pressure in the mean amounted to 50.7 atmospheres. The following table gives the precise data. The capacity used was 77.3 meters, the primary current 6.2 amperes, the gas pressure 607 mm of mercury.

TABLE II.

Length of spark mm	Throw in mm vaseline oil	Pressure in the spark in atmospheres
1	85	41
1.5	120	57
2	182	51
2.5	227	45
3	260	59
3.5	280	47
4	300	55

§5. The effect of the pressure of the surrounding medium was also investigated. It was obviously necessary to employ mercury as the index liquid in this case. The results follow. We employed a spark gap of 3 mm, a capacity of 100.2 meters, a primary current of 9.5 amperes.

TABLE III.

Pressure in mm of mercury	Throw in mm of mercury	Pressure in spark in atmospheres
585	7.8	27.2
550	6.2	19.6
502	5.0	19.9
415	3.3	11.2
320	2.2	6.5
217	1.2	3.7
96	0.5	1.0

The brass rods of 3 mm diameter were employed here and in all the following series of observations, where mention is not expressly made of other electrodes.

It should be said that at the pressures of 217 and 96 mm of mercury the electrodes were enveloped in blue light. As it was not feasible to follow the phenomenon at still lower pressures on account of the small throws at such a short spark gap, we increased it to 24 mm, and obtained the following table.

TABLE IV.

Pressure in mm of mercury	Throw in mm of mercury
165	12.3
160	11.1
140	8.8
120	7.1
88	4.7
70	3.5
50	2.8
20	1.0

We have not determined the pressure, as the assumptions made in the derivation of the formula in §2 are no longer sufficiently satisfied, and it would be very difficult to make and to interpret the photographs. The aureole which develops is so faint that it could not be made visible on instantaneous exposures, while the blue light enveloping the electrodes, as well as the path of the sparks, is so actinic that time exposures which would show the aureole would be entirely fogged by the diffuse light from the spark. Moreover the blue light at the electrodes is very unsteady, so that even an approximate measurement of the photograph would be hopeless even if the above difficulties could have been overcome.

§6. We also made a test of the effect of the shape of the electrodes by replacing the brass rods by brass balls of 18 mm diameter, the conditions of the experiment being the same as for Table III. The results were identical with those given above.

§7. On the contrary we could detect an influence of the surrounding gas on the spark-pressure. Air, carbon dioxide and illuminating gas (of density 0.47) were investigated, with a spark gap of 2 mm, a capacity of 72.8 meters, a primary current of 6 amperes, and a pressure of 704 mm of mercury in the gas enclosed in the sphere.

TABLE V.

	Throw in mm of vaseline oil	Pressure in spark in atmospheres
Air - - - - -	167	51.7
Carbon dioxide - -	323	153.2
Illuminating gas - -	260	79.7

§ 8. Experiments were further made with electrodes of different materials. We employed for these the transformer and condensers which are in use in the spectroscopic researches of F. Exner and E. Haschek, in order to obtain, as far as possible, the same experimental conditions as theirs. We also determined the spark pressure for brass electrodes for purposes of comparison with our previous figures. The carbon rods were cut from commercial pressed gas carbon, the large proportion of easily decomposed and volatile constituents of which may well explain the high value of the spark pressure.

The conditions of the experiments were — a primary current of 8.2 amperes and six batteries of ten-plate, plane condensers with a total capacity of 750 m with some 5800 volts effective pressure in the secondary. The following table gives the direct throw, the pressure, and the length of the spark, which could not be kept the same for all the electrodes.

TABLE VI.

Substance	Spark gap in mm	Throw in mm of vaseline oil	Pressure in spark in atmospheres
Gas carbon	3	740	124
Iron	3	605	79
Brass	3	427	64
Zinc	2.6	243	44
Copper	2.5	187	33

The spark pressures here found for the different metals may perhaps be brought into relation with the counter-electromotive forces measured by V. v. Lang[1] in the constant current arc. The substances which furnish a large value of the spark pressure show a high counter-electromotive force, zinc only being an exception. This may, however, be explained by the fact that the value found for the electromotive force of the zinc arc may be affected by considerable uncertainty on account of its easy melting.

§ 9. From what has been adduced in § 2 an examination of the dependence of our phenomena upon the number of sparks

[1] *Wiener Berichte*, **95**, 84, 1887.

per second seems desirable. As such an investigation could naturally not be conducted with the transformer, since we were held to the number of alternations of the current furnished by the "Internationalen Elektricitätsgesellschaft," we employed a Ruhmkorff coil of about 15 cm sparking distance. The interruptions of the primary current were made for the slower frequency with a regulating Foucault interrupter, for the higher frequency with a Neef hammer, or with a rapid motor contact-breaker. A capacity of 5.16 m was included in the secondary circuit. The dependence of the throw upon the number of interruptions may be seen from the table.

TABLE VII.

Number of interruptions per second	Throw in mm of vaseline oil
4.8	2.3
7.5	4.3
14.1	11.2
20.0	14.7
24.0	15.2
30.0	15.5

With the capacity employed a spark followed each interruption. Since now for a change from 24 to 30 interruptions per second the throw increased by only 0.3 mm of vaseline oil, or about 2 per cent. of the throw at 24 sparks per second, it is clear that with a slightly greater number of interruptions—which we unfortunately could not reach—the limit mentioned in § 2 would have been reached. It follows from this, however, that for the transformer with 80 alternations per second, the pressure given in the tables, which is primarily only a time mean, coincides with the true spark pressure.

We also attempted to obtain a measure of the pressure prevailing in the spark of an induction coil. But here a difficulty is at once encountered. Since the spark proved to be too faint for an instantaneous photograph when only slight capacity was included in the circuit, while with larger capacity the number of sparks per second must not pass a certain maximum, we were

obliged to make our measures at a low number of interruptions and then to reduce them to the highest number by means of the above table. In this way the spark pressure came out as 11.3 atmospheres for a capacity of 16.4 meters and a directly observed throw of 11.5 mm of vaseline oil.

§10. On the basis of the view developed in §2, the question could be decided whether we have to do with a continuous or a disruptive process in the direct current arc, since an elevation of pressure manifests itself here in an entirely analogous way. If we started an arc of say 2 mm length in our globe, the vaseline oil fell at first rapidly and uniformly; if the arc was then suddenly cut off, the fall not only ceased at the same moment but as suddenly changed to a rise. Now, the index liquid indicates the partial pressure in the gas at the place where the manometer tube is attached to the globe. If this increased pressure at that place is not renewed by successive waves of condensation, the manometer will remain at rest until the expanding metallic vapor has itself reached the manometer tube. In view of the large volume of our globe and of the character of the whole phenomenon, a satisfactory explanation can only be given when we suppose that there is also in the arc an elevation of pressure which continually renews itself in a pulsating manner. This assumption is open to less objection by reason of experimental researches made elsewhere that indicate such an intermittence of the discharge.[1] We may also infer the existence of the pressure thus caused from the experimental fact that the spectra of the same element obtained in different arcs display very considerable differences, which could be simply explained by the difference of the pressure prevailing in the different arcs.

By analogy with the values found above for spark-gaps we should estimate this pressure to be from two to three atmospheres for an arc of 3 mm length and a current of about 2 amperes and 110 volts.

In conclusion, we beg to briefly revert to the result of this paper. It is the demonstration of a high pressure in the path of

[1] See LECHER, *Sitzungsberichte*, **95**, 1897; CANTOR, *Sitzungsberichte*, **107**, 1898.

he electric spark, which may be considered to be due to the metallic vapor thrown off from the electrodes. This explains how the pressure in the spark varies with the energy expended, as well as with the substance of the electrodes. A further result is the dependence of the phenomenon upon the pressure and nature of the surrounding gas. Finally, the arc light also shows a rise of pressure which points to an intermittence of the discharge.

MINOR CONTRIBUTIONS AND NOTES.

THE YERKES OBSERVATORY OF THE UNIVERSITY OF CHICAGO.

BULLETIN NO. 10.

PERIOD AND ELONGATION DISTANCE OF THE FIFTH SATELLITE OF JUPITER.

THE fifth satellite of Jupiter was measured by Professor Barnard with the 40-inch telescope on five nights in March and April 1898. The elongation distances and the times of elongation have been determined from the series of measures which proved to be best adapted for these purposes. The results for the several dates are:

TIMES OF EAST ELONGATION.[1]

1898, March, $2^d\ 12^h\ 57.80^m$
1898, March, 6 12 36.11

EAST ELONGATION DISTANCES.

1898, March 6, 48.14″ (Δ=5.20)
1898, April 5, 48.12 (Δ=5.20)

So far during the present year the satellite has been observed by Professor Barnard on four nights, two of which were especially favorable for the determination of both the elongation time and the elongation distance. The results for these dates are given below:

TIMES OF EAST ELONGATION.

1899, April, $25^d\ 13^h\ 5\ .26^m$
1899, May, 1 12 32.72

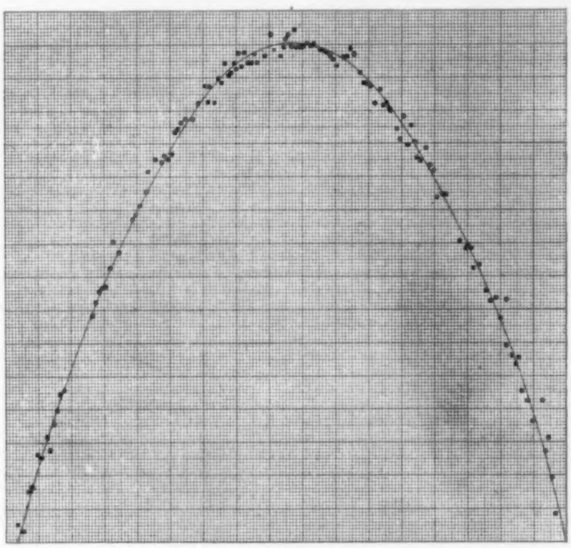
EAST ELONGATION DISTANCES.

1899, April 25, 48.34″ (Δ=5.20)
1899, May 1, 48.29 (Δ=5.20)

[1] The recorded times are six hours slow of Greenwich.

On both occasions the satellite was very well seen; 124 measures were made on April 25 and 130 on May 1. The different values of the elongation distance are due to the revolution of the line of apsides, which, as Tisserand has shown, takes place in a period of five months.

The consistency with which measures of this difficult object can be made with the great telescope is shown by the diagram, on which is



MEASURES OF THE FIFTH SATELLITE.
May 1, 1899.

platted every individual measure made on May 1. The large squares represent single seconds of arc (ordinates) and ten-minute intervals of time (abscissae). Each dot in the figure stands for a single setting of the wires, and none of the measures are excluded.

The great number of revolutions of the satellite which have occurred since its discovery in 1892 enables its period to be determined with great accuracy. Using the observed east elongation of September 10, 1892, and the east elongations of March 6, 1898, April 25, 1899, and May 1, 1899, the following values result for the periodic time of the satellite:

PERIOD OF THE FIFTH SATELLITE.

		Revolutions
1892, September 10 to March 6, 1898,	$P = 11^h\ 57^m\ 22.652^s$	4020
1892, September 10 to April 25, 1899,	11 57 22.637	4853
1892, September 10 to May 1, 1899,	11 57 22.653	4865
Mean - - - - - -	$11^h\ 57^m\ 22.647^s$	

The resulting mean period can hardly be in error more than 0.01^s. This would correspond to an error in the prediction of the satellite's position of something like a minute of time in ten years.

These observations, like many others which Professor Barnard has made, clearly illustrate the advantages which result from the use of the 40-inch telescope in the measurement of faint and difficult objects.

GEORGE E. HALE.

May 13, 1899.

BULLETIN NO. 11.

HEAT RADIATION OF THE STARS.

IN an important paper published in 1890[1] Professor C. V. Boys describes his unsuccessful attempts to detect heat radiations from the stars by means of an exceedingly delicate radiomicrometer used in conjunction with a 16-inch reflecting telescope. In spite of the fact that his apparatus was sensitive enough to show the heat equivalent to that of a candle 1.71 miles away, no effect whatever could be obtained from Venus, Jupiter, Saturn, Mars, Arcturus, Capella, Vega, or any of the numerous bright stars observed. After discussing the earlier papers of Huggins[2] and Stone,[3] Professor Boys concludes that the heating effects of stars, obtained many years ago by these observers with comparatively insensitive apparatus, were spurious.

The work of Dr. E. F. Nichols, Professor of Physics in Dartmouth College, in perfecting the radiometer, and adapting it for the measurement of heat radiations, has placed astrophysicists in possession of an instrument which for certain purposes is superior to the radiomicrometer, bolometer, or most improved form of thermopile. In view of the remarkable sensitiveness of the radiometer and its suitability for stellar work, Professor Nichols was invited to make an attempt to detect

[1] *Proceedings of the Royal Society*, **47**, 480, 1890.

[2] *Ibid.*, **17**, 309, 1869. [3] *Ibid.*, **18**, 159, 1870.

stellar heat radiations with its aid at the Yerkes Observatory. The investigation was accordingly undertaken in July 1898.

The experiments were made in the heliostat room of the Yerkes Observatory, where the radiometer, stably mounted upon a heavy pier, could be shielded from air currents and other sources of disturbance. The great steadiness of the reflected image of the scale, making it possible to record deflections to tenths of a millimeter, was doubtless due to this arrangement.

The radiometer, constructed by Professor Nichols especially for these experiments, essentially consists of a suspension system formed of two mica disks, each 2 mm in diameter, blackened on one face, and supported by a light cross-arm on either side of a thin glass staff, hung by an exceedingly fine quartz fiber in a partial vacuum. Both vanes were exposed to the radiation of the sky, at the focus of a silvered glass mirror of 24 inches aperture and 8 feet focus, made by Mr. G. W. Ritchey, Optician of the Yerkes Observatory. Rays from the star were reflected into the concave mirror by means of a siderostat[1] having a large plane mirror of silvered glass. After reflection at the concave mirror and also at the surface of a small flat fixed at an angle of 45° with the optical axis, the rays entered the radiometer through a fluorite window.

With this apparatus a deflection of 0.1 mm would be given by a candle 15 miles away, assuming total reflection at the silvered surfaces and neglecting atmospheric absorption. When the Moon's image is made to fall on one of the vanes, the scale is instantly thrown out of the field of view. Professor Nichols' radiometer is about five times as sensitive as Boy's radiomicrometer, and the area of his telescope mirror is 2.4 times that of the mirror used by Boys. In Professor Nichol's apparatus there is, however, one additional reflection.

Seven determinations of the heat radiation of Arcturus, made on August 4, 5, 7, 8, 9, 11, and 13, give a mean deflection of 0.60 mm. Each evening's determination is the result of from 21 to 47 deflections, and the probable error of the corresponding means ranges from 0.08 mm to 0.17 mm. Vega was also observed on seven nights, and gave a mean deflection of 0.27 mm. The ratio of the heat radiation of Arcturus to that of Vega, determined on five nights, is 2.1, 2.0, 3.0, 2.3, 1.0.[2] Mean 2.1. These results are not corrected for atmospheric absorption.

[1] Kindly loaned by the Allegheny Observatory.

[2] Sky very hazy.

In all cases the observer was ignorant of the probable direction of the deflection, and other precautions were taken to avoid bias. The results appear to be trustworthy, and the probable errors are not greater than might be expected in such observations. In view of the smallness of the deflections, and the uncertainty which arises from rapid fluctuations in the atmosphere, Professor Nichols does not greatly rely upon the quantitative value of the results. They may fairly be considered to show, however, that we do not receive from Arcturus more heat than would reach us from a candle at a distance of five or six miles, no account being taken in the latter case of atmospheric absorption.

GEORGE E. HALE.

MAY 17, 1899.

THE NEW ALLEGHENY OBSERVATORY.

A MOVEMENT to secure the erection of a new building, and to supply an adequate instrumental equipment for the Allegheny Observatory, was inaugurated by Mr. J. A. Brashear over a year ago. Numerous subscriptions were made by friends of the Observatory, and the fund, although not yet complete, has grown to such proportions as to insure the success of the plan. Professor F. L. O. Wadsworth, until recently a member of the staff of the Yerkes Observatory, has been elected to the directorship. Plans for the building, which embody many novel and ingenious features, have been prepared by Professor Wadsworth, and are now in the hands of the architect. Special provision will be made for astrophysical investigations, which will form the principal work of the Observatory. The largest instrument will be a refracting telescope of 30 inches aperture, with object-glass by Brashear. A full account of the new Observatory will be published in a future number of this JOURNAL.

G. E. H.

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VOLUME IX NUMBER 5

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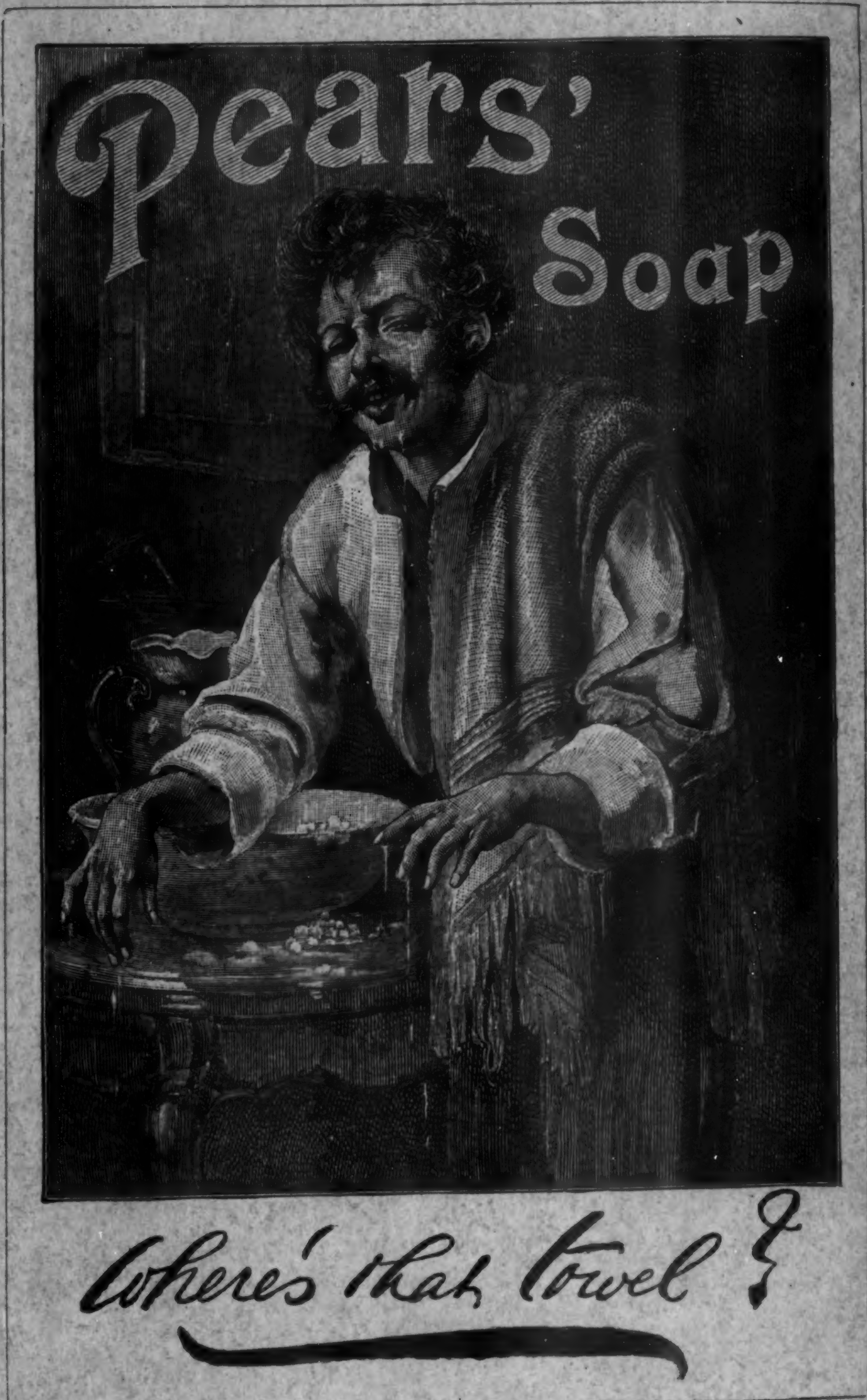
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THE

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AND ASTRONOMICAL PHYSICS

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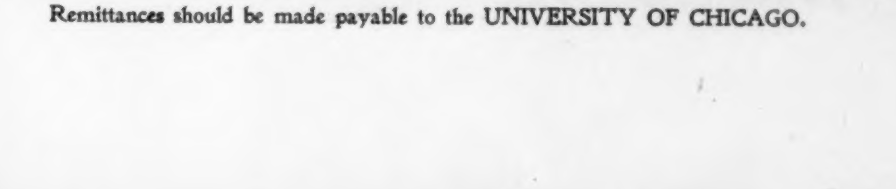

Astronomical Photographs

On account of the frequent requests received at the Yerkes Observatory for lantern slides and prints from astronomical photographs, it has been thought advisable to make provision for supplying the very considerable demand. Mr. G. Willis Ritchey, Optician of the Observatory, who has had wide experience in making and copying astronomical negatives, has undertaken to furnish such photographs at moderate expense. He is prepared to supply lantern slides, transparencies, and paper prints from any of the negatives in the collection of the Yerkes Observatory.

Among the subjects available at the present time may be mentioned: Professor Hale's photographs of prominences, faculæ and other solar phenomena, and of stellar spectra; Professor Barnard's portrait-lens photographs of the Milky Way, nebulæ, comets, and meteors; Professor Burnham's photographs of the Moon, Winter and Summer views of Mt. Hamilton and the Lick Observatory; Mr. Ellerman's photographs of the buildings and instruments of the Yerkes Observatory; and Mr. Ritchey's Kenwood Observatory photographs of the Moon.

A more complete list of subjects may be had on application to G. WILLIS RITCHEY, Yerkes Observatory, Williams Bay, Wisconsin, *to whom all orders should be addressed.*


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